BOOK REVIEWS

Foundations of algebraic topology. By S. Eilenberg and N. Steenrod. Princeton University Press, 1952 (second printing, 1957). 15+328 pp. $7.50.

This book has had a profound influence on the development of topology both before and after its publication. In the five years since its first printing it has become a standard textbook and reference work for anyone interested in topology.

The first course in algebraic topology is usually a difficult one for the student. He faces a mass of unfamiliar algebraic machinery whose motivation is difficult to grasp and whose applicability is appreciated only much later. Realizing this, the authors have adopted an axiomatic approach to the subject of homology theory. Starting with seven easily stated axioms relating algebra and geometry (and assuming only the basic concepts of algebra and point set topology as prerequisites) they show how many important and interesting theorems can be proved directly from these axioms. The axioms themselves are presented without motivation, but their immediate application is intended to make it easier for the student to accept them. Only after the reader has seen the power of the theory is he led into the details of the existence and uniqueness of homology theories.

In order to state the axioms the concept of *admissible category* is introduced. This is a family of pairs (X, A) of topological spaces and continuous maps $f: (X, A)\to(Y, B)$ between them which, roughly speaking, contains sufficiently many pairs and maps to state the axioms. Then a *homology theory* on such an admissible category consists of three functions. The first is a function which assigns to every pair (X, A) in the category and every integer q an abelian group $H_q(X, A)$. The second function assigns to every map $f: (X, A)\to(Y, B)$ in the category and every integer q a homomorphism $f_*: H_q(X, A) \to H_q(Y, B)$. The third function assigns to every pair (X, A) in the category and every integer q a homomorphism $\partial: H_q(X, A)\to H_{q-1}(A)$ (where, in the latter group, the pair $(A, 0)$ has been abbreviated to A).

The three functions H_q, f_*, ∂ of a homology theory are required to satisfy seven axioms. The first three assert the functorial (or naturality) properties of f_* and ∂. The others are: the *exactness axiom*, which relates the homology groups of (X, A), X, and A in an exact sequence; the *homotopy axiom*, which asserts that homotopic maps induce the same homomorphism; the *excision axiom*, which asserts that $H_q(X, A)$ depends, to a great extent, only on $X-A$; and the