ON ISOMORPHISMS OF GROUP ALGEBRAS

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With every locally compact topological group G there is associated its group algebra $L(G)$, the space of all complex Haar-integrable functions on G with convolution as multiplication. Considerable work has been done toward discovering the extent to which the algebraic structure of $L(G)$ determines G (see [**1**; **2**; **5**]), but some very specific questions have been left unanswered. For instance: Is the group algebra of the circle isomorphic to that of the torus? The theorem announced here stems from this question.

THEOREM. *The group algebra of a locally compact topological group T is isomorphic to that of the circle group C if and only if T is a direct sum $C+F$, where F is a finite abelian group.*

The proof leans heavily on that of Theorem 1 of [**4**]. In the outline below we will mainly be concerned with pointing out the changes in [**4**] which are needed to yield the stated result.

If $L(T)$ and $L(C)$ are isomorphic, then T is abelian, and the dual group Γ of T is homeomorphic to J, the group of all integers (the dual group of C) [**2**, p. 478]. Thus Γ is discrete and countable, and T is a compact abelian group with countable base.

Abelian groups will be written additively; for $x\in T$ and $\phi\in\Gamma$ the symbol (x, ϕ) will stand for the value of the character ϕ at the point x; the Haar measure on T will be denoted by m.

LEMMA 1. *Corresponding to every $E\subset T$ with $m(E)>0$, there is only a finite set of characters ϕ such that, for all $x\in E$,*

$$|1-(x,\phi)| < 1. \tag{1}$$

Note that (1) holds if and only if the real part of (x, ϕ) exceeds 1/2. If f is the characteristic function of E and if ϕ satisfies (1), then $|\int_T(x, \phi)f(x)dx| > m(E)/2$, and the lemma follows from the Bessel inequality.

LEMMA 2. *Every infinite subset A of Γ contains an infinite subset B, such that for some $x\in T$ the inequality*

$$|1-(x,\phi)| \geqq 1 \tag{2}$$

holds for every $\phi\in B$.

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