RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

ON 2-SPHERES IN 3-MANIFOLDS

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1. Let M be a connected, orientable, triangulated 3-manifold and let $\Lambda \subset \pi_2(M)$ be a sub-group which is invariant under the operators in $\pi_1(M)$. By a $\pi_1(M)$-class in $\pi_2(M)$ we mean the set of elements $\pm\xi a$, for some $a \in \pi_2(M)$ and every $\xi \in \pi_1(M)$. The invariance under $\pi_1(M)$ means that, if $a \in \Lambda$, then the entire $\pi_1(M)$-class $\{\pm\xi a\}$ is contained in Λ. A $\pi_1(M)$-class is represented by a map $S^2 \to M$, without reference to base-points or orientation. We shall describe such a map, or singular sphere, as *essential* mod Λ if, and only if, the corresponding $\pi_1(M)$-class is not contained in Λ. The terms polyhedral, piecewise linear etc., when applied to M, will refer to the piecewise affine structure which M derives from the given triangulation K. Thus a polyhedron in M is the carrier of a subcomplex, L, of a (rectilinear) subdivision of K. The polyhedron is compact if, and only if, L is a finite complex.

The main purpose of this note is to show how the proof of the (qualified) sphere theorem, due to C. D. Papakyriakopoulos [**5**], can be modified so as to yield a proof of:

THEOREM (1.1). *If* $\Lambda \neq \pi_2(M)$, *then* M *contains a non singular polyhedral* 2-*sphere which is essential* mod Λ.

On taking $\Lambda = 0$ in (1.1) we obtain the sphere theorem in full generality.

By attaching 3-cells to M we can imbed it in a space X such that Λ is the kernel of the injection $\pi_2(M) \to \pi_2(X)$. Hence it follows that (1.1) is equivalent to:

THEOREM (1.2). *If* $M \subset X$, *where* X *is a topological space, and if there is a map* $S^2 \to M$, *which is essential in* X, *then* M *contains a nonsingular, polyhedral* 2-*sphere which is essential in* X.

In particular if X is any orientable 3-manifold, which need not be paracompact, and if f: $S^2 \to X$ is an essential map, then every neighbourhood in X of fS^2 contains a nonsingular 2-sphere, which is