JOHN VON NEUMANN'S WORK IN THE THEORY OF GAMES AND MATHEMATICAL ECONOMICS

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Of the many areas of mathematics shaped by his genius, none shows more clearly the influence of John von Neumann than the Theory of Games. This modern approach to problems of competition and cooperation was given a broad foundation in his superlative paper of 1928 [**A**].[1] In scope and youthful vigor this work can be compared only to his papers of the same period on the axioms of set theory and the mathematical foundations of quantum mechanics. A decade later, when the Austrian economist Oskar Morgenstern came to Princeton, von Neumann's interest in the theory was reawakened. The result of their active and intensive collaboration during the early years of World War II was the treatise *Theory of games and economic behavior* [**D**], in which the basic structure of the 1928 paper is elaborated and extended. Together, the paper and treatise contain a remarkably complete outline of the subject as we know it today, and every writer in the field draws in some measure upon concepts which were there united into a coherent theory.

The crucial innovation of von Neumann, which was to be both the keystone of his Theory of Games and the central theme of his later research in the area, was the assertion and proof of the Minimax Theorem. Ideas of pure and randomized strategies had been introduced earlier, especially by Émile Borel [**3**]. However, these efforts were restricted either to individual examples or, at best, to zero-sum two-person games with skew-symmetric payoff matrices. To paraphrase his own opinion expressed in [**J**], von Neumann did not view the mere desire to mathematize strategic concepts and the straight formal definition of a pure strategy as the main agenda of an "initiator" in the field, but felt that there was nothing worth publishing until the Minimax Theorem was proved.

As the *leitmotiv* of this article, the Minimax Theorem requires at least informal statement at the outset. (A later section will present

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[1] There are two bibliographies at the end of this article, one of von Neumann's work in the field and the second for other references. Letters in square brackets refer to the von Neumann bibliography and numbers in square brackets refer to the other bibliography.