

VON NEUMANN'S CONTRIBUTIONS TO QUANTUM THEORY

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That von Neumann has been "par excellence" the mathematician of quantum mechanics is as obvious to every physicist now as it was a quarter of a century ago. Quantum mechanics was very fortunate indeed to attract, in the very first years after its discovery in 1925, the interest of a mathematical genius of von Neumann's stature. As a result, the mathematical framework of the theory was developed and the formal aspects of its entirely novel rules of interpretation were analyzed by one single man in two years time (1927–1929). Conversely, one could almost say in reciprocity, quantum mechanics introduced von Neumann into a field of mathematical investigation, operator theory, in which he achieved some of his most prominent successes.

Von Neumann's major contributions to quantum mechanics are his development of the mathematical framework of the theory and his formal study of quantum statistics, quantum measuring processes and their interrelations. Whereas the latter study was essentially complete in 1927 (except for the quantum ergodic theorem of 1929) the work on the mathematical foundations of quantum mechanics came to its culmination in 1929 with the spectral theorem for hypermaximal symmetric operators in Hilbert space. In the next two paragraphs we shall discuss these major contributions.

The mathematical framework of quantum theory. By the time von Neumann started his investigations on the formal framework of quantum mechanics this theory was known in two different mathematical formulations: the "matrix mechanics" of Heisenberg, Born and Jordan, and the "wave mechanics" of Schrödinger. The mathematical equivalence of these formulations had been established by Schrödinger, and they had both been embedded as special cases in a general formalism, often called "transformation theory," developed by Dirac and Jordan. This formalism, however, was rather clumsy and it was hampered by its reliance upon ill-defined mathematical objects, the famous delta-functions of Dirac and their derivatives. Although von Neumann himself attempted at first, in collaboration with Hilbert and Nordheim [1], to edify the quantum-mechanical

Received by the editors December 23, 1957.