
It follows, e.g. from Burnside's basis theorem (see e.g. P. Hall, Proc. London Math. Soc. vol. 36 (1932) p. 35), that in a finite $p$-group $G$ with derived group $G' \neq 1$, the quotient group $G/G'$ cannot be cyclic. It was shown by O. Taussky (J. London Math. Soc. vol. 12 (1937)) that for 2-groups with $G/G'$ of type $(2,2)$ the derived group $G'$ is cyclic and hence that $G'' = 1$. It is a difficult problem to estimate the length of the chain of successive commutator subgroups if the type of $G/G'$ is given. The next cases to be studied are the types $(2,4)$, $(2,2,2)$ and $(3,3)$. (See also W. Magnus, Math. Ann. vol. 111 (1935) and N. Ito, Nagoya Math. J. vol. 1, 1950.) (Received December 27, 1957.)


Not every matrix of determinant 1 with integral rational elements is a commutator of matrices of the same nature. (See L. K. Hua and I. Reiner, Trans. Amer. Math. Soc. vol. 71 (1951).) What are necessary and sufficient conditions for a matrix of determinant 1 with rational integral elements to be a commutator of (1) integral matrices with determinant \pm 1, (2) integral matrices with determinant 1. (Received December 27, 1957.)


A number of similar theorems are known for matrices with positive elements (positive matrices) and for positive definite symmetric matrices, but for which the available proofs are different. Can a unified treatment be given for both cases? Four examples of such theorems are:

1. The dominant eigenvalue exceeds the diagonal elements.

2. The intervals spanned by the quotients $\sum k a_{ik}/x_i$ for a positive vector $x_1, \ldots, x_n$ include the dominant eigenvalue of a positive matrix; the intervals spanned by the quotients $\sum k a_{ik}/x_i$ for an arbitrary non-null vector include an eigenvalue of a symmetric matrix.

3. The inequality

$$\det a_{ik} \geq \det a_{i1} \cdot \ldots \cdot a_{in} \cdot (a_{ik})$$

for matrices with non-negative minors of all orders and for positive definite symmetric matrices. (This was pointed out to K. Fan who already found a unified treatment).

4. A matrix with all its minors of all orders non-negative has all eigenvalues real and non-negative; a positive semi-definite symmetric matrix has all eigenvalues real and non-negative. (Received December 27, 1957.)