RESEARCH PROBLEMS

9. Olga Taussky.

It follows, e.g. from Burnside's basis theorem (see e.g. P. Hall, Proc. London Math. Soc. vol. 36 (1932) p. 35), that in a finite p-group G with derived group $G' \neq 1$, the quotient group G/G' cannot be cyclic. It was shown by O. Taussky (J. London Math. Soc. vol. 12 (1937)) that for 2-groups with G/G' of type (2,2) the derived group G' is cyclic and hence that $G''=1$. It is a difficult problem to estimate the length of the chain of successive commutator subgroups if the type of G/G' is given. The next cases to be studied are the types (2,4), (2,2,2) and (3,3). (See also W. Magnus, Math. Ann. vol. 111 (1935) and N. Itô, Nagoya Math. J. vol. 1, 1950.) (Received December 27, 1957.)

10. Olga Taussky.

Not every matrix of determinant 1 with integral rational elements is a commutator of matrices of the same nature. (See L. K. Hua and I. Reiner, Trans. Amer. Math. Soc. vol. 71 (1951).) What are necessary and sufficient conditions for a matrix of determinant 1 with rational integral elements to be a commutator of (1) integral matrices with determinant ± 1, (2) integral matrices with determinant 1. (Received December 27, 1957.)

11. Olga Taussky.

A number of similar theorems are known for matrices with positive elements (positive matrices) and for positive definite symmetric matrices, but for which the available proofs are different. Can a unified treatment be given for both cases? Four examples of such theorems are:

1. The dominant eigenvalue exceeds the diagonal elements.

2. The intervals spanned by the quotients $\sum_k a_{ik}/x_i$ for a positive vector $x_1, \cdots, x_n$ include the dominant eigenvalue of a positive matrix; the intervals spanned by the quotients $\sum_k a_{ik}/x_i$ for an arbitrary non-null vector include an eigenvalue of a symmetric matrix.

3. The inequality

$$\det_{i,k=1,\cdots,n}(a_{ik}) \leqq \det_{i,k=1,\cdots,p}(a_{ik}) \cdot \det_{i,k=p+1,\cdots,n}(a_{ik})$$

for matrices with non-negative minors of all orders and for positive definite symmetric matrices. (This was pointed out to K. Fan who already found a unified treatment).

4. A matrix with all its minors of all orders non-negative has all eigenvalues real and non-negative; a positive semi-definite symmetric matrix has all eigenvalues real and non-negative. (Received December 27, 1957.)