

denotes the n th prime, then $p_{n+1} < 2p_n$. Legendre conjectured, but no one has ever proved, that $p_{n+1} - p_n < p_n^{1/2}$ for all sufficiently large n . Hoheisel in 1930 established the existence of a number a , $1 - 1/33000 < a < 1$, such that $p_{n+1} - p_n < p_n^a$. The exponent a was successively diminished by Heilbronn in 1933, by Tschudakoff in 1936 and by Ingham in 1937. Ingham obtained $a = 5/8$ and also a somewhat smaller value. The proofs of these and related results make use of the theory of the density of the zeros of the zeta function. In Chapter Nine the machinery of this theory is developed. Applications are given also to the work of Linnik (1943, 1945), Rodosskii (1949), Tatzuza (1950) and Haselgrove (1951) on the distribution of primes in "short" arithmetic progressions. Further applications concern the estimation of $\zeta(1/2 + it, w)$.

The crowning achievement of the last chapter is the deep theorem of Linnik: Let $k \geq 2$, $(l, k) = 1$, $l < k$, and let $p_1(k, l)$ be the smallest prime in the arithmetic progression $nk + l$, $n = 1, 2, \dots$. Then there exists a constant C independent of k such that $p_1(k, l) < k^C$. The awe-inspiring proof involves forty pages and twenty-one lemmas.

The book closes with an Appendix. This contains a brief summary of pertinent theorems and formulae from the theory of functions.

The author is to be congratulated for having written an important and valuable book. The House of Springer is to be congratulated on a superb example of the art of mathematical printing.

ALBERT LEON WHITEMAN

Neue topologische Methoden in der algebraischen Geometrie. By F. Hirzebruch. *Ergebnisse der Mathematik und ihrer Grenzgebiete*, New Series, vol. 9. Springer, 1956. 165 pp. DM 30.80.

This book, devoted to the topological transcendental theory of algebraic varieties over the complex field, should rank with Lefschetz's *L'Analyse situs et la géométrie algébrique*, Paris, 1924, and Hodge's *Harmonic integrals*, Cambridge, 1941, as a milestone in the development of the theory. While topology plays the essential rôle in Lefschetz's book and Hodge's main tool is harmonic differential forms, this book is characterized by the diversity of deep and difficult results which the author drew for his use. These include, among others, Todd's genus, Thom's algebra, and Kodaira's work on complex manifolds. Sheaves (or stacks or faisceau in French and Garbe in German) and analytic bundles with their characteristic classes are the pillars on which the main result is built.

The main result, which is not proved until the very end of the book, is the Riemann-Roch Theorem for nonsingular complex alge-