

condition  $2k \equiv 0 \pmod{q-1}$  implies that  $2^{2k} - 2 \equiv -1 \pmod{q}$ .) Thus we have proved:

LEMMA 2. *The number  $2(2k-1)! s_k$ , when expressed as a fraction in lowest terms, has denominator  $b_k k_1$ .*

PROOF OF THEOREM 2. If  $r = 4k$  is a multiple of  $2(q-1)q^i$ , it follows that  $q$  is of rank  $k$ . Hence  $q$  divides  $b_k$  and  $q^i$  divides  $k_1$ ; so that  $q^{i+1}$  divides the denominator of  $b_k k_1$ . Together with Theorem 1, Corollary 2 this completes the proof.

## THE CARTESIAN PRODUCT OF A CERTAIN NONMANIFOLD AND A LINE IS $E^4$

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An upper semicontinuous decomposition  $G$  of  $E^3$  into points and tame arcs is defined in [1] such that the decomposition space  $B$  is topologically different from  $E^3$ . Interesting properties of this space have also been given by Fort [4], Curtis [2; 3], and Wilder [3]. We show that the cartesian product of the space  $B$  and a line  $E^1$  is topologically  $E^4$ . Perhaps the argument used is related to that employed by Arnold Shapiro to show that the cartesian product of a manifold described by Whitehead in [5] and a line is topologically  $E^4$ .

The arcs of the decomposition  $G$  are intersections of double tori as shown in the figure. The solid double torus contains four double tori  $T_1, T_2, T_3, T_4$  as shown; each  $T_i$  in turn contains four double tori  $T_{i1}, T_{i2}, T_{i3}, T_{i4}$  (not shown) imbedded in  $T_i$  as  $T_1, T_2, T_3, T_4$  were imbedded in  $T$ ; more double tori are imbedded in the  $T_{ij}$ 's; etc. The tame arcs of the decomposition  $G$  are the components of

$$T \cdot \Sigma T_i \cdot \Sigma T_{ij} \cdot \Sigma T_{ijk} \cdot \dots$$

Although these tame arcs are mutually exclusive, it is not possible to get a 2-sphere in  $E^3$  that misses their sum and separates two of them. No topological cube in  $T$  contains  $T_1 + T_2 + T_3 + T_4$ .

When the cartesian product is taken, the extra dimension enables one to unravel certain linking handles in the sense that if  $[a, b]$  is

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