

shown, with considerably more difficulty, that in the case  $\alpha_1 = \dots = \alpha_n = 0$ , the additional hypothesis that  $\psi$  is monotone decreasing guarantees that the inequalities in question have infinitely many solutions for almost no or almost all sets  $(\theta_1, \dots, \theta_n)$ , according as the above series converges or diverges. This hypothesis is weaker than that in the similar theorem of Khintchine.

In the final chapter the Pisot-Vijayaraghavan (PV) numbers are studied. These are the algebraic integers  $\alpha > 1$  all of whose conjugates except  $\alpha$  itself lie in the open disk  $|z| < 1$ . It is easy to see, by considering the trace of  $\alpha^n$ , that  $\|\alpha^n\|$  approaches zero as  $n \rightarrow \infty$  if  $\alpha$  is a PV number. Pisot showed, conversely, that if  $\alpha > 1$  is algebraic and  $\lambda \neq 0$  is real, and if  $\|\lambda\alpha^n\| \rightarrow 0$ , then  $\alpha$  is a PV number. Moreover, he showed that if  $\alpha > 1$  is real, and if  $\sum \|\lambda\alpha^n\|^2 < \infty$ , then  $\alpha$  is algebraic and therefore a PV number. (It is an open question whether  $\|\lambda\alpha^n\| \rightarrow 0$  implies that  $\alpha$  is algebraic.) Salem obtained the unexpected result that the set of PV numbers is closed. Proofs are given here of these three theorems; they are much simpler than the original proofs, although similar in conception.

The book closes with three appendices giving necessary tools from linear algebra and geometry of numbers, and a bibliography of papers mentioned.

W. J. LEVEQUE

*Irrational numbers.* By Ivan Niven. Carus Monograph no. 11: New York, Wiley, 1956. xii+164. \$3.00.

This most recent in a series of distinguished monographs is outstanding in organization, in clarity, and in choice of material. The book, which begins with "the preponderance of irrationals," and which closes, in Chapter X, with a proof of the Gelfond-Schneider theorem, is an admirable fulfillment of the author's purpose: "an exposition of some central results on irrational numbers . . . the main emphasis [being] on those aspects . . . commonly associated with number theory and Diophantine approximations."

The topics are arranged, in general, in order of difficulty, with the result that some of the theorems in the early part of the book are subsumed under stronger theorems later. This organization seems to have real pedagogical value. The same sort of organization is followed, to some extent, within each chapter; for example, a theorem on the uniform distribution of a sequence of irrationals is first proved by use of results on continued fractions (one of the few cases where appeal is made to the material of an earlier chapter), and then ob-