BOOK REVIEWS

An introduction to Diophantine approximation. By J. W. S. Cassels. Cambridge Tracts in Mathematics and Mathematical Physics, no. 35, New York, Cambridge University Press, 1957. 10+166 pp. $4.00.

The theory of Diophantine approximation is concerned with the approximate solution of Diophantine equations having no exact solutions, or only trivial ones. The first such problem considered was probably that of finding the good rational approximations to a real irrational number θ, or what is almost the same thing, of finding integers x, y, not both zero, for which $|\theta x-y|$ is small. This problem was generalized in the early 1840's by Dirichlet, who considered $|\theta_1 x_1+\cdots+\theta_n x_n-y|$, and about 1850 by Hermite, who considered the system of quantities $|\theta_1 x-y_1|, \cdots, |\theta_n x-y_n|$. These papers, together with that of Liouville (1844) on the existence of transcendental numbers, might be regarded as the beginnings of the subject. Much later, Minkowski suggested the term "Diophantine approximation," and used it as title for the first book (1907) in the subject.

In 1936 J. F. Koksma's *Diophantische Approximationen* appeared in the Ergebnisse series. This beautiful work describes, usually without proofs, all the important research up to the time of publication, and contains an exhaustive bibliography. It is badly out of date now, of course, because the subject is one of the most active branches of number theory. A present day bibliography would probably contain at least twice as many references as the approximately 900 in Koksma's book. Strangely enough, the additional references would be nearly exclusively to papers by European and Russian authors; it is almost as if the subject were nonexistent as far as American mathematics is concerned.

Since Koksma's book, no general work has appeared until the tract under review, a fact which further increases the value of what is, in its own right, a very important piece of work. Cassels' book does not have the scope of Koksma's, and indeed it could not have under the space limitations imposed in the Cambridge series. No attempt is made to present a complete bibliography, nor to discuss all the important problems and topics in the field. (For example, just one paragraph is devoted to transcendental numbers.) Rather, the author presents a relatively small number of theorems, each with complete proof. Many of the most beautiful and significant results of the past 20 years are to be found, in most cases with new or simplified proofs