RESEARCH PROBLEMS

1. Paul Slepian: *Problems on polynomials.*

(1) Let $0<A<1$. Let B be the set of all positive integers n such that there exist n positive numbers $a_1, a_2, \cdots, a_n$ such that the polynomial

$$(x^2 - 2Ax + 1) \prod_{i=1}^{n} (x + a_i)$$

has all non-negative coefficients. It is known that B is nonempty. (See P. M. Lewis, *The concept of the one in voltage transfer synthesis*, IRE Trans. Vol. CT-2, pp. 316–319, December, 1955.) Find the least element of B.

(2) Let $0<A<1$ and let N be the smallest integer in B, as described in (1) above. Does there exist $b>0$ such that

$$(x^2 - 2Ax + 1)(x + b)^N$$

has all non-negative coefficients?

(3) Generalize the questions raised in (1) and (2) above to the case where the factor $x^2-2Ax+1$ is replaced by an arbitrary real polynomial, say $\sum_{i=0}^{m} C_i x^i$, having no positive real roots. (Received September 13, 1957.)

2. Louis Weinberg: *Decomposition of Hurwitz polynomials.*

Let $q(s)=\sum_{k=0}^{n} a_k s^k$ represent a Hurwitz polynomial with real coefficients, i.e., all of its zeros have negative real parts. Can $q(s)$ be divided into the arithmetic sum of two polynomials,

$$q(s) = q_1(s) + q_2(s)$$

each of which has positive coefficients and only nonpositive real roots? This can easily be done in particular cases; for example, if $q(s)=(s^2+2s+5)(s+4)=s^3+6s^2+13s+20$, then $q_1(s)=s^3+6s^2+11s+6=(s+1)(s+2)(s+3)$ and $q_2(s)=2s+14$. If this can be shown to be impossible in the general case, can the decomposition always be carried out with three polynomials,

$$q(s) = q_1(s) + q_2(s) + q_3(s),$$

each of which again has positive coefficients and only nonpositive real roots? (Received September 19, 1957.)

3. R. E. Bellman: *Number theory.* I.

The problem of generating the integer solutions of the equation $x^2+y^2=1 \pmod p$ by means of the formula $x_n=\cos n\theta$, $y_n=\sin n\theta$, where (x_1, y_1) is a fundamental solution which we can write symbolically in the form $x_1=\cos\theta_1$, $y_1=\sin\theta_1$, has been extensively studied. What are the corresponding results for the equations $x_1^2+x_2^2+\cdots x_n^2=1 \bmod p$?

In particular, for the equation $x_1^2+x_2^2+x_3^2=1 \bmod p$, what subset of solutions do we obtain by means of the formulas

$$x_1 = \cos k\theta_1 \cos l\theta_2,$$
$$x_2 = \cos k\theta_1 \sin l\theta_2,$$
$$x_3 = \sin k\theta_1,$$

where $k, l=0, 1, \cdots$, and θ_1, θ_2 correspond to certain primitive solutions? (Received May 22, 1957.)