BOOK REVIEWS

Mathematical logic. By R. L. Goodstein. Leicester, Leicester University Press, 1957. 8+104 pp. 21s.

In the preface, the author states that his aim is "to introduce teachers of mathematics to some of the remarkable results . . . in mathematical logic during the past twenty-five years." He goes on to say that the book is designed for mathematicians with little or no previous knowledge of symbolic logic and is largely self contained in that proofs of major results are given in detail. He concludes "a great many different facets of the subject have been briefly sketched, but rigour has not been intentionally sacrificed for ease of reading."

This short book covers a wide range of topics. In the Introduction (10 pp.) the Frege-Russell definition of number serves both as an illustration of a representative concern of logic and as an opportunity for introducing the reader to customary logical notations. The necessity of the class concept in treating cardinal numbers is indicated. The formalistic calculus-of-numerals approach to number theory is also discussed, and the observation is made that it is different in level rather than correctness.

Chapter I (17 pp.) on *sentence calculus* begins with truth-table validity. The author's use of numerical representing functions at this point is engaging and instructive. The Russell-Whitehead axiomatization is then described and a deduction theorem outlined. Completeness (not proved), consistency, independence (proved for one axiom), three-valued logic, bracket-free notation (with Łukasiewicz's axioms) are discussed, a natural inference formulation is set forth and asserted to be complete. The Heyting intuitionist system is given.

Chapter II (16 pp.) on (lower) *predicate calculus*, gives both an axiomatic formulation with deduction theorem and Quine's natural inference formulation. Validity, satisfiability, and decision for finite domains and monadic calculus are presented. The Gödel completeness theorem is reached through Henkin's proof of the Skolem-Löwenheim theorem.

Chapter III (29 pp.) on *number theory* presents first the Hilbert-Bernays system Z and then R. Robinson's finitely axiomatizable subsystem Z_f. The theory of primitive recursive functions is briefly developed and use of the Chinese remainder theorem in showing primitive recursive functions to be arithmetical is outlined. (Numeral-wise expressibility is stated but not proved.) More general kinds of recursion are described, and reductions to primitive recursion from