BOOK REVIEWS

Einführung in die Operative Logik und Mathematik by Paul Lorenzen, Berlin, Göttingen, Heidelberg, Springer-Verlag, 1955. 298 pp.

This book presents in more detail than earlier articles an important and original approach to the foundations of logic and of mathematics. Both disciplines are reinterpreted to have as subject matter operations according to schematic rules. Hence the name *operative*. For convenience, only rules for operating on symbol strings are considered, each list of such (primitive) rules determining a formal system (*Kalkül*). A (well-formed) rule, metarule, metametarule, . . . of a formal system K can be written in the form $A_1, \cdots, A_n \rightarrow A$, where enough dots are to be added above $\rightarrow$ to indicate the composition, where $\rightarrow$ is to be omitted when $n=0$, and where $A_1, \cdots, A_n, A$ are respectively formulas (i.e., strings consisting of atoms of K and of variables ranging over the atom strings of K), rules, metarules, . . . of K. It is *admissible in K* if respectively the derivable atom strings, admissible rules, admissible metarules, . . . of K are closed under it. *Consequential logic* is then a theory of *universal* admissibility, i.e., of forms of rules, metarules, . . . which are admissible in any system K. For one after another of the symbols $\wedge$, $\vee$, $\bigwedge_x$, $\bigvee_x$, $\neg$, $\equiv$, and ι_x admissibility in K (or a suitable extension of K) of rules, metarules, . . . containing these symbols is then defined, the technical device used depending on the desired formal properties of the symbol introduced. Intuitionistic and also classical systems of logic for the corresponding symbols are then likewise theories of universal admissibility, each theorem being the form of a universally admissible rule, metarule, ... For any K, x ranges here over the *objects* of K, given as the theorems of some K', $\equiv$ indicates similar shape of objects, and ι_x serves as description operator in forming *terms* with properties similar to objects.

Terms serving as abstractions are introduced in a similar manner. If $\rho(x, y)$ is an *abstract equality relation* in K in the sense that $\rho(x, x)$ and $\rho(x, z) \wedge \rho(y, z) \rightarrow \rho(x, y)$ are admissible in K, then admissibility in K is defined for those rules, metarules, . . . containing $\iota_y^\rho \rho(X, y)$ which satisfy a certain requirement of *compatibility* with ρ. $\iota_y^\rho \rho(X, y)$ might ordinarily be regarded as a name for the class of symbol strings equivalent under ρ to X, but here it is treated *as* a class, Cantor's notion of class being discarded as too vague. In particular, if "=" is some abstract equality relation in K, if $X(z)$ is an object form in the sense that substitution of objects for z yields objects, if $y(z)$ ranges