LOCALLY TAME CURVES AND SURFACES IN THREE-DIMENSIONAL MANIFOLDS[1]

O. G. HARROLD, JR.

1. Introduction. The program to be described here is concerned primarily with an imbedding problem in the topology of 3-manifolds. A preliminary remark or two will relate this problem with those of a more general nature.

The compact 1-manifold without boundary, i.e. the simple closed curve, has long been characterized both as a subset of the plane and as an abstract space. The Jordan curve theorem and its converse, due to Schoenflies, accomplish the former. The latter is accomplished, for instance, by the theorem of Wilder [29][2] that among the locally compact, locally connected continua, the simple closed curve is distinguished by the fact that for any pair of points A and B, the continuum is a union of two irreducibly connected sets from A to B having in common only these points.

The compact orientable 2-manifolds without boundary likewise admit characterizations both as subsets of three-space, say, and as abstract spaces. The first step in the direction of the former was taken by Brouwer around 1912 and completed by Wilder in 1930. For citations to the rather extensive literature relating to the latter we refer to van Kampen's article [21]. Specifically, however, it may be pointed out that among the Peano spaces, the topological 2-sphere is characterized elegantly as the set satisfying the Jordan curve theorem nonvacuously. Zippin established the result in this form and gave an analogous characterization of the closed 2-cell [31]. Had the first characterization of the 2-sphere as an abstract space followed its characterization as a subset of three-space, one would be tempted to feel that perhaps a characterization of the 3-cell or 3-sphere along the lines of Zippin's work for dimension 2 would await the characterization of the 3-cell, say, as a subset of three-space. There are two characterizations of the 3-sphere or 3-cell as abstract spaces published to date that we are aware of. One is due to Bing [5], the other due to Woodard [30]. Bing has used his characterization in the solution of several problems.

An address delivered before the Lexington Meeting of the Society on December 1, 1956, by invitation of the Committee to Select Hour Speakers for Southeastern Sectional meetings; received by the editors December 11, 1956.

[1] This paper was supported in part by the National Science Foundation, G-2793.

[2] Numbers in brackets refer to the bibliography at the end of the paper.