

BOOK REVIEWS

Linear inequalities and related systems. Ed. by H. W. Kuhn and A. W. Tucker. Annals of Mathematics Studies, no. 38, Princeton, Princeton University Press, 1956. 22 + 322 pp. \$5.00.

This is the first collection of papers devoted to a subject standing midway between the study of convex sets in linear spaces which it uses and that of certain optimization problems, including minimax approximation, linear programming and matrix games, to which it is applied, and which in turn bear on decision processes in statistical and economic theory. Delineated by Fourier in connection with mechanical equilibrium as well as minimax error problems, given (at the occasion of determining a fundamental region for the set of all positive definite quadratic forms under a discrete transformation group) its substance and essential results by Minkowski, developed by Farkas, Dines, Schoenberg, Weyl and others and systematically established in the reviewer's inaugural dissertation, the theory of linear inequalities with its ramifications has, in the last decade, attracted increasing attention in this country, and also in the U.S.S.R., while modern high-speed computing has made its practical application feasible.

The present study contains nine papers dealing with general linear systems and their geometric and extremizational counterparts, followed by nine papers on special cases. Of the latter, those by Dantzig, Fulkerson, Heller, Hoffman, Kruskal, Kuhn, and Tompkins consider systems whose basic solutions belong to the ring (not merely field) of coefficients, in particular array problems; the paper by Gale shows that, in $2n$ -space, convex polyhedra with an arbitrary number of vertices exist every n of which form a face (cf. Motzkin, *Comonotone curves and polyhedra*, Bull. Amer. Math. Soc., vol. 63 (1957), p. 35); Kuhn and Gale re-prove the theorems of Wald and von Neumann on economic equilibrium; a related economic problem is brought into game-theoretic form by Thompson and shown to have only a finite number of solutions.

Of the papers on general systems, Fan's is the first systematic study of linear inequalities which includes infinite systems, systems in infinite dimensional linear spaces and also systems in complex linear spaces. Duffin, *Infinite programs*, likewise deals with linear operators. A new algorithm for linear programs is given by Dantzig, Ford and Fulkerson. Wolfe considers matrix games with polyhedral, instead of simplicial, strategy sets. Mills establishes an elegant theorem on