ALGEBRAIC GROUP-VARIETIES

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1. Definitions and preliminaries. Let k be an algebraically closed field, and let S be an n-dimensional projective space over k; the *points* of S are the ordered sets $\{\xi_0, \cdots, \xi_n\}$ of elements of k, with the exception of the set $\{0, \cdots, 0\}$, and where two sets $\{\xi\}$, $\{\xi'\}$ are identified if $\xi_i' = \rho\xi_i$ for each i, and for some $\rho \in k$. An (algebraic) *variety over* k is the set V of all the points $\{\xi\}$ whose coordinates $\xi_0, \cdots, \xi_n$ satisfy a given finite set of homogeneous algebraic equations with coefficients in k; V is *irreducible* if it is not the join of two nonempty varieties, neither of which contains the other. The restriction to algebraically closed fields is not strictly necessary, but is very convenient for expository purposes, and will be kept in force throughout this address. The case in which k is the complex field will be referred to as *the classical case.*

The (*cartesian*) *product* $V \times F$ of two varieties is defined in the usual way; it is a (pseudo-) variety embedded in a biprojective space, but it is also birationally equivalent (see below), in a one-to-one manner, to a variety (the *Segre product*) in the previous sense. A *cycle on* V is an element of the free abelian group generated by the subvarieties of V; a cycle is *effective* if all its (irreducible) *components* appear in it with a positive coefficient (*multiplicity*). If V, F are irreducible, an effective cycle D on $V \times F$ is also called an *algebraic correspondence between* F *and* V; to a point $P \in F$ corresponds the variety $D[P]$ consisting of all the $Q \in V$ such that $P \times Q \in D$; if P is *generic* (that is, if it does not belong to a certain proper subvariety of F), and if all the components of D have the same dimension and operate on the whole F, a certain multiplicity $e(U)$ can be attached to each component U of $D[P]$; the cycle $\sum_U e(U)U$ on V is then denoted by $D\{P\}$ (see [1] and [2] in the bibliography at the end of the paper).

The algebraic correspondence D between F and V is called a *rational mapping of* F *into* V if $D\{P\}$ is a point, with multiplicity 1, of V for a generic $P \in F$; if D is a rational mapping of F into V and also of V into F, it is called a *birational transformation*, and F, V are then said to be *birationally equivalent* [31]. These definitions, and this language, are used in [1; 2; 3; 4; 5; 6; 7; 8; 31; 32]; quite different,

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