A PARTITION CALCULUS IN SET THEORY

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1. Introduction. Dedekind's pigeon-hole principle, also known as the box argument or the chest of drawers argument (Schubfachprinzip) can be described, rather vaguely, as follows. *If sufficiently many objects are distributed over not too many classes, then at least one class contains many of these objects.* In 1930 F. P. Ramsey [12] discovered a remarkable extension of this principle which, in its simplest form, can be stated as follows. *Let S be the set of all positive integers and suppose that all unordered pairs of distinct elements of S are distributed over two classes. Then there exists an infinite subset A of S such that all pairs of elements of A belong to the same class.* As is well known, Dedekind's principle is the central step in many investigations. Similarly, Ramsey's theorem has proved itself a useful and versatile tool in mathematical arguments of most diverse character. The object of the present paper is to investigate a number of analogues and extensions of Ramsey's theorem. We shall replace the sets S and A by sets of a more general kind and the unordered pairs, as is the case already in the theorem proved by Ramsey, by systems of any fixed number r of elements of S. Instead of an unordered set S we consider an ordered set of a given order type, and we stipulate that the set A is to be of a prescribed order type. Instead of two classes we admit any finite or infinite number of classes. Further extension will be explained in §§2, 8 and 9.

The investigation centres round what we call *partition relations* connecting given cardinal numbers or order types and in each given case the problem arises of deciding whether a particular partition relation is true or false. It appears that a large number of seemingly unrelated arguments in set theory are, in fact, concerned with just such a problem. It might therefore be of interest to study such relations for their own sake and to build up a *partition calculus* which might serve as a new and unifying principle in set theory.

In some cases we have been able to find best possible partition relations, in one sense or another. In other cases the methods available to the authors do not seem to lead anywhere near the ultimate

Part of this paper was material from an address delivered by P. Erdös under the title *Combinatorial problems in set theory* before the New York meeting of the Society on October 24, 1953, by invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings; received by the editors May 17, 1955.