

## BOOK REVIEWS

*Transfinite Zahlen.* By H. Bachmann. (Ergebnisse der Mathematik und ihrer Grenzgebiete, New Series, vol. 1.) Berlin, Springer, 1955. 7+204 pp. 29.80 DM.

This first volume in a new series of the *Ergebnisse* continues the admirable tradition of the original series, of presenting a very valuable up-to-date guide to the results, problems, and literature of a special field—in this case the theory of transfinite numbers: ordinal numbers and powers, but not general order types. The author develops the theory from the beginning, in a clear and rigorous fashion, and presents proofs of most of the theorems cited, so that the book could well serve as the basis for an intensive course in transfinite numbers. The treatment is founded on the Zermelo-Fraenkel axiom system but is expressed in the language of naïve set theory, and although axiomatic questions are considered, the emphasis is definitely on the arithmetical aspects of the theory. It is shown how far it is possible to get in various branches of the subject before applying the axiom of choice to derive further results, and the use of this axiom is always expressly indicated.

The first chapter opens with a brief discussion of naïve set theory, the foundations problem, and the various schools of mathematics. The axioms are then introduced, as well as the fundamental notions of equivalence, similarity, well-ordering, transfinite induction, and transfinite number.

The next two chapters are devoted to the theory of ordinal numbers, mostly without the use of the axiom of choice. One finds here, in addition to standard material, a simple treatment of principal numbers, as well as recent results concerning normal functions, regressive functions, sequences of continuous functions, rarified classes of ordinal numbers, and permutations of sequences of ordinal numbers.

The fourth chapter is concerned with the arithmetic of powers without the aid of the axiom of choice, and then in the fifth chapter this axiom is used to develop the arithmetic of cardinal numbers. Much of what is known concerning alephs and beths (transfinite powers of alephs) is to be found here. The generalized continuum hypothesis, some consequences of it, and some propositions equivalent to it are also discussed.

Chapter six deals with the well-ordering and the cardinal number