AN OUTLINE OF THE THEORY OF PSEUDOANALYTIC FUNCTIONS[1]

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1. **Introduction.** Pseudoanalytic functions are, roughly speaking, solutions of generalized Cauchy-Riemann equations. Such functions were considered by Picard [67; 68] and by Beltrami [3; 4], but the first significant result was obtained by Carleman [28] in 1933, and a systematic theory was formulated only recently. No comprehensive account is available in print and the main results have been announced only episodically and, in most cases, under unnecessarily restrictive hypotheses. In this paper we shall outline the main features of this theory.

The theory of analytic functions of a complex variable occupies a central place in analysis and it is not surprising that mathematical literature abounds in generalizations. In some generalizations one extends the domain of the functions considered, or their range, or both (functions of several complex variables, analytic functions with values in a vector space or an algebra, analytic functions of hypercomplex variables, analytic operators, etc.). If we restrict ourselves to functions from plane domains to plane domains, or, more generally, from Riemann surfaces to Riemann surfaces, we encounter two well known and very useful generalizations of analytic functions: interior functions and quasi-conformal functions. Interior functions [47; 60; 61; 62; 63; 78; 86] have all topological properties of analytic functions and no others. As a matter of fact, they may be defined as functions which can be made analytic by a homeomorphism of the domain of definition. Quasi-conformal functions [2; 27; 46; 66; 75; 79] are interior functions subject to an additional metric condition.[2] If the functions are assumed to be continuously differentiable mappings, this additional condition requires that infinitesimal circles be taken into infinitesimal ellipses of uniformly bounded eccentricity.

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[2] Some authors, to the present writer's regret, use the term pseudoanalytic for quasi-conformal functions.