BOOK REVIEWS

Begründung der Funktionentheorie auf alten und neuen Wegen. By L. Heffter. Berlin, Springer, 1955. 8+63 pp. 12.60 DM.

The "foundations of function-theory" are understood here to consist of the study of the minimal possible set of assumptions under which a complex function $f(z)$ can be expanded into a power series. Generally such assumptions fall into three broad categories: the existence and continuity of the derivative (Cauchy), the unique existence of the derivative (Goursat), and the unique integrability of $f(z)$ (Morera-Osgood). In the present monograph, the author pictures the foundations of the theory as consisting of six distinct categories with the equivalence of the Cauchy and Weierstrass theories forming one such category, the Cauchy-Goursat theory a second, and the Morera-Osgood theory a third, which is asserted to be " . . . *nur aus historischem Interesse.*" Into a fourth category falls the Looman-Menchoff Theorem and the theory associated with it. The fifth and sixth categories were devised by the author himself in 1936 and 1951, respectively, and each, in its way, can be considered as a variation or extension of the idea of unique integrability of $f(z)$. More specifically, the fifth category consists of showing that if, in a domain G, the real and imaginary parts of $f(z)$ satisfy Cauchy-Riemann difference equations [in terms of mean values of integrals] over every rectangle in G with sides parallel to the axes, then $f(z)$ is regular in G, while the sixth consists of showing that every integral of $f(z)/(z-\zeta)$ vanishes over a certain class of closed paths in G [the point ζ must lie outside the closed path].

Except for the Looman-Menchoff Theorem, the book is completely self-contained, beginning with the notion of the convergence of a sequence and of a Dedekind cut, and proceeding, with all necessary proofs, to Gauss' theorem and the usual properties of functions of a complex variable needed at this stage. All line integrals which are considered are taken either over polygonal paths with segments parallel to one of the axes, or over arcs of circles, a fact which does not compromise generality, but which, in this instance, permits a clearer presentation of the underlying principles. The only theorem which is not proved is the full Looman-Menchoff Theorem which is stated on page 38 together with something of its background; however, a diluted form of the theorem is stated and proved.

The introductory material and the development of the six categories comprise two chapters; in a third (and last) chapter there is a selected list of original papers in chronological order together with the