ASYMPTOTIC PHENOMENA IN MATHEMATICAL PHYSICS

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The problems I intend to speak about belong to the somewhat undefined and disputed region at the border between mathematics and physics. The fields of physics from which these problems originate are rather classical; the mathematical questions involved are also rather classical. That does not mean that these problems belong to the past. On the contrary, they are quite alive today and—I am convinced—they will remain so for some time.

The problems concern what may be called asymptotic phenomena. Instead of explaining in general terms what I mean by asymptotic phenomena, I prefer to single out at first one class of such phenomena: *discontinuities*. A typical discontinuity of the kind I have in mind is the boundary of the *shadow* which appears when a light wave passes an object. Now, the propagation of light is governed by a partial differential equation which has continuous solutions. How then is it possible that a discontinuity arises? Of course, actually there is no sharp discontinuity at the shadow boundary; there is a transition from light to dark which takes place across a very narrow strip along the shadow boundary. Nevertheless, it is remarkable enough that the differential equations of wave motion have solutions which involve such quick transitions—in fact, most differential equations of physics possess such solutions—and it is an interesting task to study those features of these equations which make such quick transitions possible.

Discontinuities and quick transitions occur in various branches of physics. A striking example of a discontinuity is the *shock* in gas motion. Quick transitions occur frequently in situations in which one perhaps would not speak of a discontinuity. A case in point is Prandtl's ingenious conception of the *boundary layer*. This is a narrow layer along the surface of a body, traveling in a fluid, across which the flow velocity changes quickly. Prandtl's observation of this quick transition was the starting point for his theory of fluid resistance. Other cases, closely related to the boundary layer phenomenon, are the so-called *edge effect* in the deformation of elastic plates and shells and the *skin effect* in the flow of electric currents. A number of other such effects will be described in the later parts of this lecture. All

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