# TOPOLOGY OF LIE GROUPS AND CHARACTERISTIC CLASSES[1]

ARMAND BOREL

**1. Introduction.** The notion of continuous group, later called Lie group, introduced by S. Lie in the nineteenth century, has classically a local character. Although global Lie groups were also sometimes considered, it is only after 1920 that this concept was clearly formulated. We recall that a Lie group in the large is first a manifold, i.e., a topological Hausdorff space admitting a covering by open sets, each of which is homeomorphic to euclidian $n$-space; second it is a group; third it is a topological group, i.e., the product $x \cdot y$ of $x$ and $y$ and the inverse $x^{-1}$ are continuous functions of their arguments; and finally it is required that there exist coordinates in a neighbourhood $V$ of the identity element $e$ such that if $x$, $y$, and $x \cdot y$ are in $V$, the coordinates of $x \cdot y$ are analytic functions in the coordinates of $x$ and $y$. Gleason, Montgomery, and Zippin recently proved that the last condition follows from the others, thus solving Hilbert's fifth problem, with which we shall not be concerned here.

As soon as the concept was defined with precision, there arose the problem of studying topological properties of such group-manifolds. Indeed, in the first paper which systematically considers global Lie groups, H. Weyl's famous paper on linear representations [81], a key result which states that the fundamental group of a compact semi-simple Lie group is finite is topological in nature. The question was next considered by E. Cartan in several papers, and later on by many mathematicians; as a matter of fact, it was often generalized in order to include also the study of "homogeneous spaces," i.e., manifolds which admit a transitive Lie group of homeomorphisms. A very complete survey of the work done in this field up to 1951 has been published in this Bulletin by H. Samelson [67]. Although of course some overlap is unavoidable, the present report is meant as a sequel and will therefore concentrate mainly on developments which occurred during these very last years. It will be devoted for the greater