BOOK REVIEWS

Theorie der Funktionen einer reellen Veränderlichen. By I. P. Natanson. Berlin, Akademie-Verlag, 1954. 12+478 pp. 26 DM.

This is a textbook in real function theory, first published in 1941 in Russian. A revised and expanded Russian edition appeared in 1950, and it is this that has now been translated into German.

Briefly, the contents are as follows. (These phrases are the reviewer's summaries, not necessarily the author's chapter titles.) I–II: countable sets, open and closed sets on the real line. III–VI: Lebesgue measure on the real line, measurable functions, the Lebesgue integral and its properties. VII: orthogonality in Hilbert space, Fourier series. VIII: functions of bounded variation, and Stieltjes integrals. IX: absolute continuity and the differentiation of the Lebesgue integral. X: singular integrals and trigonometric series. XI–XIII: measure and integration in two dimensions, Fubini theorem, differentiation of set functions. XIV: ordinal numbers. XV: Baire classification of functions. XVI: topological properties of certain function spaces. XVII: the role played by the Russians in the development of real function theory.

The assumed prerequisite for a course based on this book is differential and integral calculus—in the European, not the American, manner. Thus in most schools in this country this would have to be a second course in real function theory. Specifically, the theory of limits is assumed understood—lim, lim sup, lim inf, and the notion of uniform convergence are not even defined. Furthermore, though certain properties of the real number system are investigated, this is not done in any systematic way. For example, there is a proof of the Bolzano-Weierstrass theorem based on the nonempty intersection of a nested set of closed intervals, this latter being referred to as a "well known theorem from the theory of limits."

The reviewer would like to suggest that regardless of what background is assumed, there are two essentially independent criteria on which a real function theory book may be classed as "advanced" or "elementary." (1) The degree of abstraction and generality in the subject matter considered. (2) The thoroughness with which investigations are prosecuted—the extent to which all facets of a problem are considered and "best possible" theorems are ferreted out. Natanson's book must be rated "elementary" on the first of these counts and "advanced" on the second.

Specifically, the only measure that appears in the book is Lebesgue