

BOOK REVIEWS

Calculus, a modern approach. By K. Menger. 2d ed. Illinois Institute of Technology Bookstore, 1953.

This book is the outcome of a long program of research by Professor Menger to simplify the ideas and notation of calculus. The most striking feature of it is the treatment of variables.

The classical variable, for which the notations of calculus were developed, was vaguely a generic symbol x which stands for the elements of a set X . This is akin to the algebraist's idea of an indeterminate. In the classical variable terminology, the symbol $f(x)$ represents a variable which is a function of the variable x . The Weierstrassian variable x was a symbol which stands for a "fixed" ("arbitrary") but unspecified element of a set X . Ordinarily a proposition involving a Weierstrassian variable is preceded by quantification. In terms of Weierstrassian variables the symbol $f(x)$ is regarded as representing the value of the function f at x .

The confusion between the symbol which represents the function and that which represents its evaluation remains a serious defect in the language of calculus to the present day, for although a writer may set out to employ a pure Weierstrassian point of view, and though all goes well with the basic topological theory, he gets into trouble with the machinery of calculus when he comes to speak of the function x^2+1 , or $\sin(n\theta+\phi)$. The trouble grows deeper when he begins to cope with substitution theory, with multiple integrals and partial derivatives.

Menger retains the Weierstrassian variable with more or less standard notation but he makes a sharp break with the classical variable x whose domain is the set X , replacing it by the identity function I on X to X . The result is not only a conceptual clarification and simplification of the ideas of calculus, but, when this program is worked out in detail in an elementary text, there appear some illuminating perspectives on the rôles of the two languages. The Weierstrassian language works well for the function theoretic foundations, but once we are in possession of the properties of continuous functions on compact intervals, the calculus proper is better served by Menger's modern substitute for the classical variable language. Essentially what happens is that the introduction of the identity function in place of a variable carries the subject upstairs into the realm of linear operators in the algebra $C(x)$ of continuous functions.