

THE STRUCTURE OF TOPOLOGICAL SEMIGROUPS

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The title of this address might incline one to the notion that here is to be found a small number of large theorems. To the contrary, I shall talk about a large number of small theorems. Actually, there does not exist at this time any corpus of information to which the title "structure of topological semigroups" is in any fashion applicable. Whether such a body of theorems will ever exist is a matter for the future and is likely to depend on the use to which it might be put as well as to the tastes of mathematicians who are not yet such.

When the investigation of topological groups began there was at hand a theory of abstract groups and much of a fundamental character in Lie groups was available. Beyond this there existed a great body of geometry even if some of it was in a nebulous state insofar as the then held standards of rigor were concerned.

With topological semigroups the situation is quite contrariwise. Here we are faced with a lack of satisfactory algebraic results. I do not think that there are so many as twenty-five papers each exceeding ten pages which are concerned exclusively with the algebraic aspects.

We are more fortunate than were the pioneers who forayed the frontiers of topological groups in that we have at our disposal a greater wealth of topology. Much that they could not—or at least did not—use is at hand for our use. Furthermore we can rely, at least if for no more than analogy, on their results. The state of both algebraic and set-theoretic topology is a somewhat happier one now than then. Still we are likely to be troubled for awhile for lack of something like Haar measure without which we shall be at a loss for representation theorems. At present there seems to be no line of attack on the representation problem and it is probable that we shall need to rely to a greater extent on geometry and topology than was the case with groups.

1. Introduction. My interest in this field began many years ago when, as a graduate student, I first learned of the beautiful theorem of E. Cartan,

If an n -sphere is a topological group then $n=0, 1$ or 3 .

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