REGULAR CONVERGENCE

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1. **Introduction.** The notion of convergence is certainly one of the most important ones in all mathematics. In analysis if a sequence of functions converges to a limit function, we ask the question what properties enjoyed by the members of the sequence are carried over to the limit function. With no restrictions on the type of convergence, of course very little can be said. We, therefore, impose conditions on the convergence which will allow some conclusions to be made. The same situation prevails in topology if we consider the concept of convergence of point sets. We say that the sequence of points sets (G_i) converges to the point set G, where all sets belong to a Hausdorff space, if the following is true. Every point with the property that each of its neighborhoods contains points from infinitely many G_i lies in G and each point of G has the property that each of its neighborhoods contains points from all but a finite number of the (G_i). It is easily seen that G will always be closed regardless of whether the G_i are or not. This notion was first introduced by Zarankiewicz [1]. An equivalent definition in a compact metric space is as follows. If we call the spherical neighborhood of a point set X with radius ϵ the set of all points x whose distance from some point in X is less than ϵ, then (G_i) converges to G if G is closed and for every ϵ the spherical neighborhood about G with radius ϵ contains all but a finite number of the G_i and the spherical neighborhood with radius ϵ about all but a finite number of the G_i contains G. For example, the sets $G_i = \{(x, y) \mid x = 1/i, (0 \leqq y \leqq 1)\}$ converge to $G = \{(x, y) \mid x = 0, (0 \leqq y \leqq 1)\}$. The second definition is equivalent to saying that G is closed and the Hausdorff distances [2] from G_i to the G converge to 0. Very few properties are carried over to the limit set by convergence of this general type. The reason for this is that two sets can be close to each other without being at all similar. For example in the above mentioned example the sets G_i could be replaced by the points of the line forming G_i that are rational with denominator i, and the limit set would still be the same. The reverse situation is not true, however, i.e. if all the members of the sequence in a compact metric space are closed and connected, then the limit set will also be closed and con-

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