five and seven. There is hardly anything in the book, for instance, on the stability of periodic solutions, or in the sixth chapter on the second order linear equation with periodic coefficients. There is comparatively little reference to work done in the last ten years either in this country or abroad. Aside from these omissions, however, Bellman's book is a pleasant and interesting contribution to the theory of differential equations.

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Discontinuous automatic control. By I. Flügge-Lotz. Princeton University Press, 1953. 8+168 pp. $5.00.

Although self-regulating devices have been in operation since the days of the governor on Watt's steam engine, it is only in recent years that the subject of automatic control has assumed a central position in the engineering and industrial world.

From the mathematical side, the control problem leads to systems of nonlinear differential equations in the following way. If we assume that the state of the physical system is specified at time t by the vector $x(t)$, the study of small displacements from equilibrium gives rise, in a system without control, to a linear vector-matrix equation $\dot{x}=Ax$. If we now consider a system with control, where the control is manifested by a forcing term and the magnitude of the control is dependent upon the state of the system, the resulting equation for x has the form $\dot{x}=Ax+f(x)$, and is, in general, nonlinear.

The term "continuous control" will be used to describe situations in which $f(x)$ is a continuous function of x. In many cases, it was found that continuous control was far too expensive to use. In place then of control devices which gave rise to forcing terms of continuous type, it was far cheaper to design control devices yielding forcing terms whose components are step-functions of x. The simplest version of this type of control system is one with a simple on-and-off control mechanism. This type of control is called "discontinuous automatic control."

A simple example of the mathematical equations which result is the following second order equation, $\ddot{u}+a\dot{u}+bu=c \operatorname{sgn} (\dot{u}+ku)$, where u is now a scalar function. This equation has the form $\ddot{u}+a\dot{u}+bu=c$, over the region of phase space described by $\dot{u}+ku>0$, and the form $\ddot{u}+a\dot{u}+bu=-c$, over the region of phase space described by $\dot{u}+ku<0$. If $\dot{u}+ku=0$, the forcing term is taken to be zero.

We observe then the interesting fact that while the equation itself is nonlinear, over the regions $\dot{u}+ku\gtrless 0$, u may be determined as a solution of a linear equation, albeit a different linear equation over different regions.