BOOK REVIEWS

Stability theory of differential equations. By Richard Bellman. New York, McGraw-Hill, 1953. 13+166 pp. $5.50.

This is the first book in the English language on the topic. In fact, while many aspects of stability theory have been summarized, notably by Russian authors, no mathematically satisfactory treatise on the subject has appeared since the classical work of Liapounoff. It is the reviewer's feeling, therefore, that any new book on stability must, at least partially, be judged on how it summarizes and organizes the highlights of the work done in this field since Liapounoff.

Bellman's book is organized in seven chapters. The first and third chapters are introductions to the theory of linear and nonlinear differential equations, respectively. Matrix and vector notation is used throughout these two well-written chapters. Chapter two deals with questions of stability, boundedness, and asymptotic behavior of linear differential equations, many of them originally investigated by Bellman himself. Chapter four is the heart of the book. Three different proofs are given for the stability theorem for the case where the linearized system has constant coefficients. The proofs are clear. A clever counterexample is provided to show that one cannot hope to extend the theorem without modification to the case where the linearized system does not have constant coefficients. In chapter five the asymptotic behavior of certain first order nonlinear equations is considered. Chapter six deals with linear second order equations. After an introduction to the more elementary properties of linear equations the author proves certain theorems about the boundedness in norm of solutions using various norms. Chapter six closes with a study of the oscillatory and asymptotic behavior of the solutions. Finally, chapter seven discusses the linear second order equation,

$$\frac{d}{dt}\left(t^{\rho}\frac{du}{dt}\right) \pm t^{\sigma}u^{\mu} = 0.$$

It is shown that certain conclusions with regard to the nature of solutions can be drawn from a knowledge of ρ, σ and μ.

The book as a whole is well-written and quite readable. There are many exercises to supplement the text. However, it seems unfortunate that the author excluded certain aspects of stability theory which, in a book dedicated to this field, seem more important than chapters