

Lagerungen in der Ebene, auf der Kugel und im Raum. By L. Fejes Tóth. (Grundlehren der mathematischen Wissenschaften, vol. 65.) Berlin, Springer, 1953. 10+197 pp. 24 DM.

This is probably the only book, in any language, devoted to the subject of "arrangements" such as packings and coverings. It is full of interesting results, many of them discovered by the author himself, and the rest collected from a great variety of sources. Yet it is not a collection of isolated theorems but develops the subject systematically. The exposition is clear, and relieved by frequent historical interludes (such as one drawing attention to Minkowski's enthusiastic remark: "Mich interessiert alles, was konvex ist!"). There are 124 beautiful figures, a five-page bibliography (including many works as recent as 1951 and 1952) and a useful index.

After a brief introduction to the theory of convex regions, the author gives a neat proof that, if two ellipsoids are polar reciprocals with respect to a unit sphere, their volumes, E and E' , satisfy

$$EE' \geq (4\pi/3)^2,$$

with equality only when the ellipsoids and sphere all have the same center. This is essentially a theorem of affine geometry: If two ellipsoids are polar reciprocals with respect to a third, the geometric mean of their volumes is greater than or equal to the volume of the third. Another affine theorem, this time in two dimensions, is that, if an n -gon contains an ellipse of area e and is contained in an ellipse of area E , then

$$e/E \leq \cos^2 \pi/n.$$

This first chapter includes also a nicely illustrated account of the regular and Archimedean solids and of the analogous tessellations; e.g., (3, 3, 4, 3, 4) is the tessellation of triangles and squares in which no two squares share a side (so that each vertex is surrounded by two triangles, a square, another triangle, and another square, as the symbol indicates). The author might well have mentioned (on p. 19) that this particular tessellation is not anomalous like (3, 3, 3, 4, 4), but can be derived from the regular tessellation (4, 4, 4, 4) in the same way that the snub cube (3, 3, 3, 3, 4) is derived from the cube (4, 4, 4) or from the octahedron (3, 3, 3, 3) (cf. Coxeter, *Regular and semi-regular polytopes*, I, Math. Zeit. vol. 46 (1940) pp. 380–407, especially p. 395).

Chapter II includes a good exposition of Blaschke's important concept of the *affine length* of a curve. It is proved that the affine circumference λ of an oval curve of area T satisfies