

*Introduction to measure and integration.* By M. E. Munroe. Cambridge, Addison-Wesley, 1953. 10+310 pp. \$7.50.

In the last years, in particular since Halmos' book on measure theory, modern theory of integration has become a regular part of the mathematics curriculum. Munroe's book is one of the first designed specifically for students. It begins with the basic concepts of real function theory and covers the by now classical part of measure theory. The organization is logical in the sense that measure, measurability, integration, and differentiation are studied in that order. The exercises are selected so as to make the text better understood as well as to suggest further study. The presentation is clear and concise.

Chapter I introduces the concepts in use throughout the book: algebra of sets, additive classes, metric spaces, limits and continuity. Chapters II and III are devoted to the investigation of measures: construction and particular types. Beginning with properties of additive set functions and their Jordan decomposition, the author proceeds to the construction of measures by means of outer, then metric outer, measures. Lebesgue-Stieltjes, Hausdorff, and Haar measures are considered in some detail. Measurable functions are introduced in Chapter IV. Operations on and approximations of these functions are carefully presented. Chapter V is devoted to integration. The definition of integrals begins with that of integrals of simple functions, then of non-negative measurable functions. The study of indefinite integrals leads to the Radon-Nikodým theorem. The Fubini theorem closes the chapter. Chapter VI, entitled convergence theorems, starts with the study of various types of convergence of measurable functions, and continues with that of  $L_p$  spaces, linear functionals on Banach spaces, orthogonal expansions in Hilbert spaces, and the mean ergodic theorem. The final Chapter VII on differentiation covers differentiation of additive set functions in euclidean spaces, metric density, and differentiation with respect to nets. The book contains also a few general concepts of probability theory.

There is no doubt that the author did a remarkable job in about three hundred pages. Yet, it seems to the reviewer that for students who are introduced for the first time to measure theory, the presentation is too abstract, too fast, and too soon. The reviewer's own teaching experience and bias would lead him to a somewhat different order of presentation. He would start with the algebra of sets and Borel sets and proceed at once to measurable functions, integration, and convergence theorems. Construction of measures and metric spaces would follow. The reviewer feels that the basic property—to be emphasized—of the family of measurable functions is its closure