# EQUIVALENCE RELATIONS IN ALGEBRAIC GEOMETRY

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**1. The cycle groups $C_s$.** An algebraic variety $V$ in $n$-dimensional complex projective space $P^{(n)}$ is obtained by equating to zero a finite number of forms $F_1(x_0, \cdots, x_n), \cdots, F_m(x_0, \cdots, x_n)$ with complex coefficients; $V$ is assumed to be nonempty. If $V$ is irreducible, that is, if $V$ is not the union of a finite number of proper subvarieties, it is possible to associate with $V$ in several ways a complex dimension $d$. For example, just as $P^{(1)}$ is topologically equivalent to a real 2-dimensional sphere, so can every $P^{(n)}$ be represented topologically by a $2n$-dimensional real complex in the sense of combinatorial topology. (See [1]; numbers in brackets refer to the references.) In this representation, $V$ goes over into an even-dimensional, connected, orientable, closed complex whose dimension is defined as $2d$. This complex is denoted by $K^{(2d)}$ and $V$ itself by $V^{(d)}$.

Consider the set $T_s$ of irreducible, $s$-dimensional subvarieties of $V^{(d)}$ for some fixed $s$, where $0 \leqq s \leqq d$. A function on $T_s$ is called integral if its value for every element of $T_s$ is a rational integer, and if the function is zero except for at most a finite number of elements of $T_s$; these functions constitute of course an additive group, denoted by $C_s$. We identify the integral function which at the elements $W_1^{(s)}, \cdots, W_h^{(s)}$ of $T_s$ assumes the values $n_1, \cdots, n_h$ and which is zero everywhere else on $T_s$ with the linear combination $n_1W_1^{(s)} + \cdots + n_hW_h^{(s)}$. Since every $W_i^{(s)}$ gives rise to a $2s$-dimensional, connected, closed, orientable subcomplex of $K^{(2d)}$, the above linear combination can be interpreted as a $2s$-dimensional cycle of $K^{(2d)}$ in the sense of topology. This fact is the reason why we call the elements of $C_s$ the $s$-dimensional cycles of $V^{(d)}$ and often consider $C_s$ as a subgroup of the $2s$-dimensional cycle group of $K^{(2d)}$. A cycle is called *effective* if, considered as a function, it never assumes a negative value; otherwise the cycle is called *virtual*. The effective cycles are clearly closed under addition but not under subtraction, and every cycle is the difference of two effective cycles.

The group $C_s$ is completely determined by the cardinal number of $T_s$, and hence its structure is of no interest. The importance of $C_s$ lies in the fact that the different aspects of the geometry of $V^{(d)}$ are most conveniently studied by means of the equivalence relations which

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