BOOK REVIEWS

Sur quelques propriétés des valeurs caractéristiques des matrices carrées. By M. Parodi. (Mémorial des Sciences Mathématiques, no. 118.) Paris, Gauthier-Villars, 1952. 64 pp. 800 fr.

This book deals in the first place with the applications of the following determinant theorem: Let $A=(a_{ik})$ be a square matrix of order n with real or complex elements and let $|a_{ii}| > \sum_{k\neq i}|a_{ik}|$ $(i=1, \cdots, n)$. It follows that the determinant $D=|a_{ik}|\neq 0$. This theorem has varied applications and is being rediscovered continually. Parodi mentions some of its discoverers and some of its generalizations without making any attempt at completeness. In particular, Price's recent improvement of Ostrowski's lower bound for $|D|$ is not mentioned.

A most important application of this determinant theorem lies in its usefulness for the bounding of characteristic roots of matrices. For, it implies that the characteristic roots of A lie inside or on the boundaries of the n circles $|a_{ii}-z| = \sum_{k\neq i}|a_{ik}|$. A number of applications and generalizations are treated, in particular that due to A. Brauer in which the n circles are replaced by $n(n-1)/2$ Cassini ovals. The references are again incomplete; in particular, the pioneering work of Gershgorin is not mentioned.

Parodi exploits a theorem of Ostrowski which deals with the variations of the elements which preserve the nonsingularity of a matrix.

The book concludes with the study of the determinantal equation of the form

$$\left| a_{ik}^{(0)} z^{n} + a_{ik}^{(1)} z^{n-1} + \cdots + a_{ik}^{(n)} \right| = 0$$

where $(a_{ik}^{(j)})$ are square matrices; the case $n=2$ which turns up in electrical networks is treated earlier and upper bounds for the real parts of the roots z are determined.

Apart from various applications to circuit theory, the book deals with general stability problems, with the reducibility of polynomials, and with methods of finding bounds for the modules of the zeros of polynomials. It contains a number of worked numerical examples of matrices of small orders.

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Complex analysis. By L. V. Ahlfors. New York, McGraw-Hill, 1953. 12+247 pp. $5.00.

In American universities the course in complex analysis is often