BOOK REVIEWS

Linear algebra and projective geometry. By R. Baer. New York, Academic Press, 1952. 8+318 pp. $6.50.

A half-century or so has elapsed since the great treatises on foundations of geometry appeared. In these works it was taken for granted that the reader could stand any amount of intricate geometrical reasoning; but the idea that geometry could be done over an arbitrary division ring was quite a novelty and was accordingly treated with respect and caution. In the present book we see how much the mathematical climate has changed. The necessity of grappling with an arbitrary division ring should be the least of the reader's worries. Nor will a lack of geometric intuition seriously impede him (it is interesting to note that there are only nineteen figures in the book, of which two are non-geometrical and the last seven are concerned with the introduction of coordinates). But a good backlog of experience with the trickery of modern algebra is recommended to any prospective reader.

Let F be a division ring, A a vector space over F, both more or less arbitrary. While it is true that the case of characteristic two often gets "cavalier treatment," possible non-commutativity of F is allowed every scope for its operations. Infinite dimensionality of A is permitted, but plays a subdued role, coming to the reader's attention mainly when considerations of duality make it impossible. With a minimum of delay the object of central interest makes its appearance: the lattice L of subspaces of A. If L_1 is a second such lattice, call a lattice isomorphism between L and L_1 a projectivity. The first fundamental theorem asserts that any projectivity is induced by a semi-linear transformation, provided the dimension of A is at least three.

A lattice isomorphism of L upon itself is an auto-projectivity. In the group of auto-projectivities we can pick out the subgroup of collineations, consisting of those auto-projectivities which can be induced by linear transformations. A pitfall awaits us here, for the same projectivity may be induced by both a linear and a semi-linear transformation, provided the automorphism for the latter is inner. This is the first of many occasions on which inner automorphisms have to be accorded special consideration. It is incidentally gratifying that perspectivities, which play such an inflated part in classical accounts, are here cut down to their proper role of being possible building blocks of collineations. The discussion moves on to the second fundamental theorem (fixing a collineation by its effect on a simplex), Pappus's theorem, and cross ratio.