extended complex plane such that for all rational functions $u(z)$ satisfying $|u(z)| \leqq 1$ on Z, the transformation $u(T)$ exists and $\|u(T)\| \leqq 1$. It is proved that unitary and symmetric transformations are characterized by the fact that they have respectively the unit circle and the real axis as spectral sets.

E. R. LORCH

Über die Klassezahl abelscher Zahlkörper. By Helmut Hasse. Berlin, Akademie, 1952.

This is a highly technical book, whose object is the derivation of a formula for the class-number h of an arbitrary absolute abelian field K and the study of this formula. Such a formula had been proved by Kummer for cyclotomic fields (i.e. fields generated by a root of unity) and in the general case by several authors (Fuchs, Beeger, Gut). The source of these formulae is of course the fact that the class number appears in the expression of the residue at $s=1$ of the zeta function $\zeta_K(s)$ of K. Using the product decomposition of ζ_K into L-series, one is reduced to the computation of the values $L(1;\chi)$ at 1 of the L-series corresponding to those characters $\chi \neq 1$ which are associated to K by class field theory. The numbers $L(1;\chi)$ appear as infinite series; the main problem is to express them in closed form, which is done by making use of Gaussian sums.

The resulting formula appears in the form $h=h_0h^*$, where h_0 is the class-number of the maximal real subfield K_0 of K, while h^*, the "second factor" of h, turns out to be an integer >0. The fact that these two factors h_0 and h^* are actually integers is not obvious from the expressions for these numbers which appear in the formula itself. One of the aims of the author is to transform these expressions in such a way as to render their arithmetic nature more apparent. This in itself would not appear so very fascinating a task: when we express the number of zeros of an analytic function in a region by a contour integral, we do not take pains to establish independently that the value of this integral is an integer. However, in the process of so doing, new properties of h_0 and h^* appear which lead to a certain number of new results on class numbers of fields.

The second chapter of the book is concerned with the transformation of the expression for h_0. Here the striving to obtain for h_0 an expression which exhibits it as an integer is not entirely successful. Two different lines of attack are followed which yield results for two different kinds of fields K. The end results of the two methods are in the following form: the product of h by some integer c is expressed as the index in the group of all units of a certain sub-group generated by