Methods of algebraic geometry. Vol. 2. By W. V. D. Hodge and D. Pedoe. Cambridge University Press, 1952. 9+394 pp. $7.50.

Having carefully laid the algebraic foundations in volume I (see Bull. Amer. Math. Soc. vol. 55 (1949) pp. 315–316), the authors first define an algebraic *variety* to be the aggregate of points determined by the vanishing of a set of homogeneous polynomials over a given field K. The ground field K is quite general except that it is not allowed to have a finite characteristic. In elementary treatments of real analytic geometry, one often finds it convenient to make temporary use of complex coordinates. Here, similarly, algebraic extensions of K are often introduced for special purposes. Great care is taken in defining a *generic* point (van der Waerden's "allgemeine Nullpunkt") of an algebraic variety, and a generic member of a system of k-spaces S_k. It is proved that a generic S_{n-d} meets an irreducible d-dimensional variety V_d in a finite number of points, each of which is a generic point of V_d. The number of points is the *order* of V_d. Much use is made of the so-called *Cayley form*, which is the "zugeordnete Form" of van der Waerden and Chow (Math. Ann. vol. 113 (1937) pp. 692–704).

Chapter XI is a thorough treatment of algebraic correspondences, following van der Waerden and Weil. The general principles are illustrated by application to two classical problems: finding the transversals of four skew lines in S_3, and reducing the general ternary cubic form to $X_0^3+X_1^3+X_2^3+6\lambda X_0X_1X_2$.

In Chapter XII, the theory of intersection leads naturally to the theory of equivalence, which is defined as follows. Two varieties V_a and V_a' on a V_n (in projective space of more than n dimensions) are said to be "equivalent in the narrow sense" if they belong to the same continuous system. Two such varieties are said to be "equivalent in the wide sense" ($V_a \equiv V_a'$) if there exists another variety V_a'' such that V_a+V_a'' and $V_a'+V_a''$ are equivalent in the narrow sense. Equivalences $U_a \equiv U_a'$ and $V_a \equiv V_a'$ imply $U_a+V_a \equiv U_a'+V_a'$, but there does not necessarily exist a variety X_a such that $U_a+X_a \equiv V_a$. The desirable group property is achieved by inventing a *virtual* variety V_a-U_a and writing

$$V_a - U_a \sim V_a' - U_a'$$

when $V_a+U_a' \equiv U_a+V_a'$. The authors prove that, when the fundamental variety V_n is a flat n-space, any variety V_a of order g is equivalent to gS_a, where S_a is a flat a-space. They then mention the generalization "which has exercised the minds of many geometers:"