SOME NOTIONS AND PROBLEMS OF GAME THEORY

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The theory of games is a relatively new branch of mathematics, which treats of situations involving conflict among rational agents. In a typical problem to which the theory is applicable, each of several people can to some extent, but only partially, influence the outcome of a certain event; no one of them by himself can determine the outcome completely; and they differ in their preferences among the various possible outcomes: the theory of games is then concerned with the problem of what each person should do in order to maximize his expectation of good.

The theory was originally developed—first by von Neumann,[1] and later by Morgenstern and von Neumann[2]—to provide a mathematical basis for economics. More recently it has been successfully applied to problems of military tactics.[3] The mathematical statisticians,[4] finally, have found that some of the fundamental notions of this theory are extremely useful for their discipline: procedures analogous to those used in the theory of games, if they do not yet tell the statistician exactly what he ought to do, have at least taught him that certain things are better left undone.

In this lecture I shall summarize the mathematical aspects of this theory. The literature of the subject is already too extensive for me to be able to cover it completely in the time at my disposal, but I hope to explain some of the important notions involved, and to indicate some of the outstanding problems.

Before going to a general description of games it is well to explain some technical terminology. In the theory of games one means by a *game*, roughly speaking, a body of rules which specify unambiguously what, under various conditions, each of certain persons, called the *players* of the game, is permitted to do, what chance devices are

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[1] See von Neumann [19]. (The numbers in brackets refer to items in the bibliography.)

[2] See von Neumann and Morgenstern [20].

[3] For a nontechnical account of the relation of the theory of games to military matters, see McDonald [15] and [16]. For a detailed analysis of an interesting problem in this domain (the problem of how an airplane should cruise in searching for a submarine) see Morse [17].

[4] For the application of the theory of games to statistical problems, see Arrow, Blackwell, and Girshick [1], and Wald [25] and [26].