BOOK REVIEWS

Arithmetical questions on algebraic varieties. By B. Segre. London, Athlone, 1951. 5+55 pp. 10s. 6d.

This book is based on three lectures given by the author in King's College, London, in March 1950. He describes it in the preface as "an account of the more important problems in which algebraic concepts and methods have proved fundamental in overcoming arithmetical difficulties, or where arithmetical notions and results have played a rôle in algebraic geometry." The exposition is throughout very condensed, as might be expected on comparing the small size of the book with the large field covered. Results are quoted largely without proof, or with only an outline of the ideas used in the proof; there is however an excellent and extensive bibliography, by means of which the reader attracted by any of the subjects touched on can have no difficulty in informing himself more fully.

The first chapter, presumably corresponding to the first lecture, deals with quadrics in an arbitrary commutative field γ. First is considered the rather exceptional case where γ has characteristic 2, and in particular where it has only two elements; here the conic is not self-dual, as all its tangents pass through a fixed point, and every line through this point is a tangent, though its point of contact may not belong to γ but to a quadratic extension of γ. Another peculiar result is that the tangent lines to a general quadric surface (in the field of order 2) form a linear congruence; the surface has a unique tangent in each of its nine points, and these are all the lines meeting a particular pair of skew lines, which are the only lines in space not meeting the quadric at all. Most of the rest of the chapter exhibits how much of the ordinary theory of quadrics in the real field remains valid for an arbitrary field, so long as the latter is not of characteristic 2; as a typical result of this kind we may quote the theorem that every quadratic form in n variables is equivalent in γ (i.e., can be transformed by a reversible linear transformation with coefficients in γ) to one of the form

$$y_1y_1' + \cdots + y_hy_h' + \phi(z_1, \cdots, z_k)$$

where the y's, y''s, and z's are some or all of the new variables (so that $2h+k\leqq n$) and the equation $\phi(z_1, \cdots, z_k)=0$ has no solution in γ except $(0, \cdots, 0)$; moreover, if two expressions in this form are equivalent in γ, the numbers h, k are the same for both (this is