PARTITIONING CONTINUOUS CURVES

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I. INTRODUCTION

1. **The continuous curve.** By a continuous curve we mean a compact, locally connected, metric continuum. For definitions of locally connected, continuum, and other terms used in this paper, see either [**18**] or [**21**]. Those who like to visualize topology concretely may wish to think of a continuous curve as a chunk out of Euclidean 3-space—one that is connected (all in one piece), one that is bounded (lies on the interior of a sphere), and one that is locally connected (nearby points belong to small connected subsets). A wad of paper, an irregularly shaped rock, or the earth itself may be considered as examples. However, our remarks about continuous curves will apply equally well to those in Euclidean spaces of all dimensions and to those in a Hilbert space.

When Jordan first introduced the term continuous curve, he defined it analytically to be the image (in the plane) of a straight line interval under a continuous transformation. It was not until over twenty years later that it was discovered that any compact locally connected metric continuum was the image of a straight line interval under a continuous transformation and conversely. This interesting and unusual discovery adds spice to the study of mathematics [**24**, p. 12]. Another interesting aspect of this discovery is that it was made independently by two mathematicians, Hahn and Mazurkiewicz. Since Peano had shown earlier that a square plus its interior is the image of a straight line interval, a continuous curve is sometimes called a Peano continuum.

In this discussion we shall be interested in the continuous curve itself and not the continuous transformation of an interval. Hence, we use the definition in the first paragraph rather than the analytic one. In this discussion we shall be interested in the structure of a continuous curve.

2. **Examples of continuous curves.** A straight line interval, a square plus its interior in the plane, and a cube plus its interior in 3-space are examples of continuous curves. In fact, any closed n-cell or n-simplex is an example. One can get a less familiar continuous

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