BOOK REVIEWS

The kernel function and conformal mapping. By Stefan Bergman. (Mathematical Surveys, vol. 5.) American Mathematical Society, New York, 1950. 8+160 pp. $4.00.

Orthogonal functions and the kernel function have become fundamental tools of the modern theory of functions; they allow one to consider the single-valued regular functions which are of integrable square over a given region as the elements of a Hilbert space.

The starting point for this development was L. Bieberbach's minimum problem connected with the Riemann mapping problem (see Rend. Circ. Mat. Palermo vol. 38 (1914)). The first researches on orthogonal functions in the complex domain originated under the influence of the Berlin school (particularly Erhard Schmidt and L. Bieberbach). Stefan Bergman, himself a descendent of this school, has intensively developed the theory of orthogonal functions and their kernels with great thoroughness during many years. He has now produced a text which provides an introduction to this now extensive field.

Chapter 1 begins with the following definition of the function class $\mathfrak{L}^2$ on which the whole theory rests: $f(z)$ is called of class $\mathfrak{L}^2$ in the given schlicht bounded region B if (1) $f(z)$ is regular and single-valued and

$$(2) \qquad \mathfrak{F}_B(f) = \iint_B |f(z)|^2 d\omega < \infty.$$

The orthonormal systems $\phi_\nu(z)$ are introduced in $\mathfrak{L}^2$, the Riesz-Fischer theorem is proved, and it is shown that there are closed orthonormal systems associated with B.

$$K(z, \bar{t}) = \sum_{\nu=1}^{\infty} \phi_\nu(z)\overline{\phi_\nu(t)}$$

is called the kernel. It is defined everywhere in B and is independent of the choice of the $\phi_\nu(z)$.

In Chapter 2 it is first shown that the uniquely determined function $f(z)$ in B, with $f(t)=1$, $t\in B$, and $\iint_B|f(z)|^2 d\omega$ a minimum, is $f(z)=K(z, \bar{t})/K(t, \bar{t})$. Then some essentially more general minimum problems are solved.

Chapter 3 introduces an invariant metric with the aid of the kernel function. This is the Bergman metric, which is a true Riemannian