TOPOLOGY OF LIE GROUPS

HANS SAMELSON

1. **Introduction.** The purpose of this address is to sketch some aspects of the subject known as topology of Lie groups, i.e., of the study of the topological characteristics (mainly those of homology and homotopy theory) of the underlying spaces of the Lie groups, and of the connections between topological and group theoretical properties. The interest in this field seems to stem from the fact that a variety of disciplines from algebra, analysis, and topology have found a very natural domain of application here; on the other hand, the topological study of Lie groups has resulted in contributions to other fields such as the theory of fibre bundles [**116**],[1] (generalized) affine connections [**20**], metric geometry [**128**], and topology (one might mention that the famous theorems of de Rham [**102**] were first formulated in this context).

In [**13**] Cartan has given a beautiful account of everything that was known about topology of Lie groups at the time; in the present paper we shall try to fill in some of the subsequent developments; some overlap is of course unavoidable (see also [**115**; **141**]). It should be said that we shall be concerned only incidentally with the general theory of topological groups; in particular we are not concerned with the developments centering around Hilbert's fifth problem, i.e., the problem of when topological groups can be proved to be Lie groups.

2. **Definitions.** We begin by recalling briefly basic definitions and a series of classical facts: To describe a Lie group G, we have first of all a manifold of some dimension n, i.e. a (separable) Hausdorff space, usually assumed connected, in which every point has a neighborhood which is homeomorphic with Euclidean n-space E^n; such a homeomorphism sets up a coordinate system in the neighborhood of the point. Secondly, the manifold carries an analytic structure: A class of coordinate systems, covering the manifold, is specified, such that wherever two of the systems overlap, one has a (real-) analytic transformation of coordinates, with nonvanishing Jacobian. It becomes then possible to introduce the concepts of (real-) analytic function,

An address delivered before the Chicago meeting of the Society, April 28, 1950, by invitation of the Committee to Select Hour Speakers for Western Sectional Meetings; received by the editors June 18, 1951.

[1] Numbers in brackets refer to the bibliography at the end of the paper.