

tion of transformation groups expressing various symmetries with the integration theory of partial differential equations is discussed.

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*Introduction à la géométrie non euclidienne par la méthode élémentaire.*

By G. Verriest. Paris, Gauthier-Villars, 1951. 8+193 pp. 1000 fr.

This exposition begins with the foundations of Euclidean geometry: Hilbert's axioms of incidence, order, congruence, and parallelism. Setting aside the axiom of parallelism, the author develops a number of theorems belonging to both Euclidean and hyperbolic geometry. He takes care to avoid any appeal to intuition; for example, he proves in detail that the sum of two supplementary angles is equal to the sum of two right angles. This part of the work, making no assumption of continuity, culminates in the "second theorem of Legendre": If, in at least one triangle, the angle-sum is equal to two right angles, the same holds for every triangle. In Chapter IV, Dedekind's axiom of continuity is used to prove the axiom of Archimedes and the "first theorem of Legendre": The angle-sum of a triangle cannot exceed two right angles. The author mentions that this was anticipated by Saccheri, and that it would not hold without continuity [Dehn, *Math. Ann.* vol. 53 (1900) pp. 436-439]. Chapter V establishes the equivalence, in the presence of the other axioms, of three possible formulations of the Euclidean axiom: Euclid's own Postulate V, the unique parallel through a given point to a given line, and the angle-sum of a triangle being equal to two right angles. It follows that all three are contradicted in hyperbolic geometry.

The next three chapters are on Hilbert's treatment of area and its connection with angle-sum. Chapter IX deals with simple properties of parallels and ultraparallels in the hyperbolic plane. On p. 142 the author omits, as being "assez compliquée," the proof of the existence of a common parallel line to two given rays [Hilbert, *Grundlagen der Geometrie*, Leipzig, 1913, p. 151; Coxeter, *Non-Euclidean geometry*, Toronto, 1947, p. 205]. He excuses the omission by declaring that this result will not be required later; but he actually uses it on p. 150 and again on p. 156, each time assuring the reader that it will not be needed any more! There is a very readable account of circles, horocycles, and hypercycles, with a short paragraph on the extension to three dimensions.

Elliptic geometry is relegated to a single chapter at the end. The statement that every two coplanar lines intersect is shown to imply that each line of the plane has one or two absolute poles. The two alternative hypotheses are carefully examined, with appropriate