ISOMORPHISMS OF INFINITE-DIMENSIONAL ANALOGUES OF THE CLASSICAL GROUPS

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1. Introduction. The classical groups referred to here are the *full linear*, *symplectic*, *orthogonal*, and *unitary* groups. These are groups of linear transformations operating on a finite-dimensional linear vector space over a (commutative) field. In order to save time, we shall not repeat the usual definitions of the groups but shall proceed directly to definitions[1] which are meaningful for the infinite- as well as the finite-dimensional case. At the same time, the condition that the coefficient domain be commutative will be relaxed and semi-linear as well as linear transformations will be admitted.

The main objectives in this address are to describe the structure of isomorphisms between two such generalized classical groups and to outline some of the methods used in studying these isomorphisms. It turns out that the isomorphisms are, roughly speaking, induced by isomorphisms between the underlying vector spaces on which the transformations act.

In the case of the full linear group in finite dimensions, this problem (for automorphisms) has been considered by Schreier and van der Waerden [12][2] when the coefficient domain is commutative, and by Dieudonné [2] when the coefficient domain is not commutative. Dieudonné has also considered the other classical groups in the finite-dimensional case. Mackey [9] has considered the problem for the multiplicative group of all bounded linear transformations with bounded inverses on an infinite-dimensional real normed linear space. The case studied by Mackey is included in our generalization of the full linear group.

The method of attack employed here is the standard one of investigating the way in which involutions are transformed by the isomorphisms plus an application of the fundamental theorem of projective geometry. However the special methods used in the investigation are refinements of methods introduced by Mackey in the paper mentioned above. The main results outlined are discussed in detail for linear transformations in [10] for the full linear case

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[1] These definitions are given in the next section.

[2] Numbers in brackets refer to the bibliography at the end of the paper.