BOOK REVIEWS

Statistical decision functions. By A. Wald. New York, Wiley, 1950. 9+179 pp. $5.00.

This book, published shortly before Wald's death in an airplane crash in India, is an outgrowth and generalization of the author's papers during the period 1939–1949 on the general theory of statistical decision. It represents a remarkable application to statistical theory of the methods and spirit of modern mathematics.

The general statistical decision problem as formulated by Wald can be described in the following terms. There is given a stochastic process $X=\{X_i\}$ and a class Ω of distributions which is known to contain the true distribution F of X as an element. Any rule δ for sampling from X and arriving at a terminal decision d^t belonging to a given class D^t is called a *decision function*. For any F and d^t there is given a function $W(F, d^t) \geqq 0$ representing the loss involved in taking the decision d^t when F is the true distribution of X. All these preliminaries go to define the *risk function* $r(F, \delta)$, which is the expected value of the loss plus that of the cost of sampling, given the decision function δ and true distribution F. The object of the statistician is to choose δ so as to minimize $r(F, \delta)$. However, the statistician controls only δ while F is, so to speak, controlled by Nature. The problem is therefore analogous to that of the zero-sum two-person game of von Neumann, with the difference that, whereas the statistician wishes to minimize $r(F, \delta)$, we can hardly say that Nature wishes to maximize it.

The problem in choosing δ arises from the fact that no δ will simultaneously minimize $r(F, \delta)$ for all F in Ω. Two alternatives are considered by Wald. (1) If F has a known prior probability distribution ξ in Ω (that is, if Nature's mixed strategy is known to the statistician), then the decision problem is solved by that δ which minimizes the average risk

$$\int_\Omega r(F, \delta) d\xi; \tag{1}$$

such a δ is called a *Bayes solution* corresponding to ξ. (2) If a prior distribution ξ of F does not exist or is not known, the statistician who, in the spirit of the theory of games, regards Nature as an opponent out to maximize $r(F, \delta)$, should adopt a *minimax solution* δ for which $\sup r(F, \delta)$ with respect to F is as small as possible. Wald himself feels that this attitude is "perhaps not unreasonable."