Coefficient regions for schlicht functions. By A. C. Schaeffer and D. C. Spencer. With a chapter on *The region of values of the derivative of a schlicht function* by Arthur Grad. (Amer. Math. Soc. Colloquium Publications, vol. 35.) New York, 1950. 14+311 pp., 2 plates. $6.00.

In the theory of schlicht functions there is considerable difference in depth between the elementary results exemplified by Koebe's distortion theorem and the advanced theory exemplified by Löwner's proof of the inequality $|a_3| \leqq 3$. Through Löwner's paper variational techniques were introduced in the coefficient problem, and since that time they have provided a more and more efficient tool. Löwner's result was at first quite isolated. To obtain similar results of a more general nature Schiffer had to overcome specific difficulties which seemed rather formidable. Later on the techniques were perfected by Schiffer himself, by the authors of the book under discussion, and by many others. The time was ripe for a systematic presentation of such methods and results, and the team of Schaeffer and Spencer has obliged us with an impressive monograph on the subject.

The historical introduction is brief and leads almost directly to the formulation of the main problem. One considers the class $\mathfrak{S}$ of normalized functions $f(z) = z + a_2z^2 + a_3z^3 + \cdots$ which are regular and schlicht in $|z| < 1$. The problem is to characterize the sequences $\{a_n\}$ which define such functions; this problem will be solved if one can determine the region V_n in $(2n-2)$-dimensional space to which the point $(a_2, a_3, \cdots, a_n)$ is confined. The most likely way to success is by determination of the boundary of V_n through the extremal properties of the corresponding functions.

This problem is not the most general and not even necessarily the most natural; it is difficult to agree that the series development has special merits in comparison with other problems of interpolation. However, the coefficient problem is certainly typical in the sense that more general problems can very probably be treated by analogous methods, and the amount of interest it derives from the fact that Bieberbach's conjecture $|a_n| \leqq n$ has neither been proved nor disproved is not negligible.

Chapter II introduces the authors' specific variational method on which most of the book is based. A rough description of the method follows. Let C be the unit circle $|z| < 1$, S the Riemann sphere over the w-plane, and $w = f(z)$ a schlicht mapping. Draw a simple analytic arc γ in C, and introduce an analytic correspondence, different from the identity, between the two edges of a slit along γ. Through identification of corresponding points a new abstractly defined Riemann sur-