

tion between the images of  $h(t)$  and  $h(g(t))$  where  $g(t)$  is a standard function such as  $t^{-1}$ ,  $t^2$ ,  $e^t$ ,  $\sinh t$ , and so on, interesting applications to hypergeometric series, the Parseval relation, again with interesting applications to Bessel function identities. Chapter XII introduces step- and discontinuous functions. This is the chapter in which the functions of number theory play such a conspicuous and welcome part. While the work is an excellent practice in operational technique, the results are also relevant for the theory of numbers. The discussion of difference equations in chapter XIII is another opportunity for deriving many relations between special functions.

Chapter XIV explains the technique of solving integral equations by means of the operational calculus. Chapter XV, on partial differential equations, is somewhat disappointing. It makes stimulating reading, the technique is explained carefully, and is illustrated by excellent examples; yet the work is more or less formal. The peculiar difficulties involved in verifying the solution are not even mentioned.

In chapter XVI the operational calculus is extended to several variables, and it is shown how this simultaneous operational calculus can be utilized for the solution of partial differential equations, and other problems.

Chapter XVII contains ten pages of general formulas and rules of the operational calculus, and is aptly labelled *Grammar*. Chapter XVIII, *Dictionary*, contains 27 pages of well-classified operational transform pairs.

The book developed from lectures given by van der Pol in 1938 and 1940, the original Dutch manuscript was prepared during the late war, and the English translation was edited by Dr. C. J. Bouwkamp. The translation deserves special praise for preserving the freshness and flavor of the original. It goes without saying that it is always clear what the authors mean, even where they do not express themselves idiomatically. Passages which may lead to misunderstandings, as for instance on p. 135 where the authors say "the exponents of  $x$  may increase" when they presumably mean "the exponents of  $x$  are assumed to increase" are very rare. A very special praise is also due to the printing of this book, one of the most beautifully printed of the recent mathematical books.

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*Existence theorems in partial differential equations.* By Dorothy L. Bernstein. (Annals of Mathematics Studies, no. 23.) Princeton University Press, 1950. 10+228 pp. \$2.50.

An enormous number of papers have been written that deal with