

with a brief account (8 pages) of finite linear spaces. There are no exercises.

The book treats with clarity and precision an astonishing amount of material, and is a very welcome addition to the literature of the subject.

LEONARD M. BLUMENTHAL

The location of critical points of analytic and harmonic functions. By J. L. Walsh. (American Mathematical Society Colloquium Publications, vol. 34.) New York, American Mathematical Society, 1950. 8+384 pp. \$6.00.

As the title indicates, the book is concerned with the critical points of analytic functions $f(z)$ of the single complex variable $z = x + iy$ and of harmonic functions $u(x, y)$ of the two real variables x and y . As is well known, a critical point of $f(z)$ means a zero of its derivative $f'(z)$, and a critical point of $u(x, y)$ means a point where both partial derivatives $\partial u/\partial x$ and $\partial u/\partial y$ vanish. The former are the points where the map by $w = f(z)$ fails to be conformal and are the multiple points of the curves $|f(z)| = \text{const.}$ and $\arg f(z) = \text{const.}$ The latter are the equilibrium points in the force field having $u(x, y)$ as force potential and are the stagnation points in the flow field having $u(x, y)$ as velocity potential. Thus the subject matter of the book is one of considerable importance in both pure and applied mathematics.

In this book the analytic functions considered are largely polynomials, rational functions, and certain periodic, entire, and meromorphic functions. The harmonic functions considered are largely Green's functions, harmonic measures, and various linear combinations of them. The interest in these functions centers about the approximate location of their critical points. The approximation is in the sense of determining minimal regions in which lie all the critical points or maximal regions in which lies no critical point.

This book not only has a unity of subject matter, but it also has very nearly a unity of method. The method is based upon the observation that, with $z - a = re^{i\theta}$ and thus $(\bar{z} - \bar{a})^{-1} = r^{-1}e^{i\theta}$, the vector $(\bar{z} - \bar{a})^{-1}$ has the direction of the line segment from the point a to the point z and a magnitude equal to the reciprocal of the length of this line segment. Accordingly, the vector $m(\bar{z} - \bar{a})^{-1}$ may be regarded as force with which a particle of mass m repels a unit particle at z ; the sum $F(z) = \sum m_k (\bar{z} - \bar{a}_k)^{-1}$ may be regarded as the resultant force upon a unit particle at z due to the system of discrete masses m_k at a_k , and the integral $J(z) = \int [\bar{z} - \bar{a}(t)]^{-1} dm(t)$ may be regarded as the resultant force at z due a continuous spread of matter. Now, it turns