BOOK REVIEWS

Sur la fonction noyau d'un domaine et ses applications dans la théorie du transformations pseudoconformes. By Stefan Bergman. (Mémorial des Sciences Mathématiques, no. 108.) Paris, Gauthier-Villars, 1948. 250 fr.

This memoir is the continuation of the book *Sur les fonctions orthogonales de plusieurs variables complexes avec les applications à la théorie des fonctions analytiques* (Interscience, 1941, and Mémorial des Sciences Mathématiques, no. 106, 1947), by the same author (see Bull. Amer. Math. Soc. vol. 48 (1942)).

The starting point for the considerations outlined in the following is the kernel function $K_G(z, \bar{t})$, $z=(z_1, z_2)$, $\bar{t}=(\bar{t}_1, \bar{t}_2)$, uniquely associated with any given domain G, which has been introduced by the author and used with ever growing success. $K_G(z, \bar{t}) = \sum_{\nu=1}^{\infty} \phi^{(\nu)}(z)\overline{\phi^{(\nu)}(t)}$, where $\{\phi^{(\nu)}\}$ represents an arbitrary system of functions which are analytic in G and complete and orthonormal over G in the L^2 metric. K is a relative invariant under analytic transformations of the four-dimensional (z_1, z_2)-space.

Chapter I contains an investigation of the behavior of $K_G(z, \bar{t})$ on the boundary of G. To this end: (1) K_G is identified as the solution of the following minimum problem: The normalized function K_G minimizes $\int_G |f|^2 d\omega_z$ ($d\omega_z$ is the four-dimensional volume element) under the condition $f(t_1, t_2)=1$, where the value $\lambda_G(t)$ of the minimum is

$$\lambda_G(t) = 1/K_G(t, \bar{t}); \tag{1}$$

(2) K_G is compared with the kernel functions of such domains, both inscribed in and circumscribed about G, that have the property that their kernel functions can be constructed explicitly.

The author lists a number of such domains and their respective kernel functions. Thus, one obtains bounds for K_G; these bounds are then particularly valuable as we approach the boundary of G. Boundary points R are classified according to the smallest number n, such that $[\rho(z)]^n K(z, \bar{z})$ has a limit as the point (z) approaches R. (Here, ρ denotes the euclidian distance from (z) to R.) If R is a point totally pseudo-convex in the sense of E. E. Levi, then $n=3$. Cases are also considered where $n=0$, 1, or 2.

The core of the entire presentation is in Chapter II. By means of minimum problems, two covariant mapping functions are assigned to every region in such a way that to every couple consisting of a