

AN EXTENSION THEORY FOR A CERTAIN CLASS OF LOOPS

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Introduction. If E is a group with a normal subgroup K one may form the quotient group $E/K \cong M$. Conversely, for preassigned groups K, M , there is the *extension* problem: to determine (in some sense) all groups E with K as normal subgroup such that $E/K \cong M$. Much progress has been made on this problem, particularly through the work of Baer [1, 2, 3]¹ and the cohomology theory of Eilenberg and MacLane [1, 2, 3]. The latter authors make it clear that insight is gained by relinquishing part of the associative law; specifically, by requiring that E be merely a *loop* such that the associative law $(e_1 e_2) e_3 = e_1 (e_2 e_3)$ holds if at least one of the e_i belongs to a distinguished subgroup of K . We take this to be K itself. It then becomes evident that the subclass of loops E consisting of the *groups* is not the only one of interest; one may consider, for example, the *Moufang* loops, in which case it seems natural to allow M also to be Moufang. Thus we approach the extension problem actually studied in the paper: M is a given loop, K is a group (not given, but with given centre G) and E is to be any loop with K as a normal subloop contained in the "associator" of E , such that $E/K \cong M$. This problem is more typical of group theory than of loop theory but is, nevertheless, a natural and significant special topic in the theory of loops.

For the sake of brevity no examples or applications are given and references to the bibliography are kept to a minimum. The Eilenberg-MacLane *kernels*, important for constructions, have been ignored. I may signal out as new: the inverse of a (noncentral) extension (§1), the specific results on central Moufang extensions (§6)² and the all-pervading functions F which generalize (even for M a group) the Eilenberg-MacLane cocycles. As indicated by Theorem 8 (§4), additional information about the functions F would probably increase our knowledge of cohomology groups.

1. Extensions. A *loop* M is a system with a multiplication such

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¹ Numbers in brackets refer to the bibliography at the end of the paper.

² Slightly weaker results on central Moufang extensions were obtained in 1946-1947 with the support of a Guggenheim Fellowship supplemented by a grant from the Wisconsin Alumni Research Foundation. (See Bull. Amer. Math. Soc. Abstract 53-1-11.)