462t. Werner Leutert: *On the convergence of approximate solutions of the heat equation to the exact solution.*

It is shown that an approximate solution of the heat equation can be obtained from a three line difference equation by using only half of the particular solutions of the form $e^{i\beta x}e^{\alpha i}$. The approximate solution will converge to the exact solution for all positive values of the mesh ratio $r=\Delta t/(\Delta x)^2$ and it will be stable in the sense that small changes in the initial condition vanish as the time t is increased. von Neumann's test shows instability for all values of $r>0$. (Received July 31, 1950.)

463t. Bertram Yood: *On fixed points for semi-groups of linear operators.*

Let G be a semi-group of bounded linear operators on a normed linear space X, and G^* be the family of adjoints of elements of G. Sets of conditions are given on G which imply the existence of a nonzero fixed element for G^* (in X^*). In particular if X is the space of bounded functions on a set S, the results show, as a special case, the existence of a finitely-additive measure defined for all subsets of S invariant under a solvable group of 1-1 transformations of S onto S. This fact is due to von Neumann (Fund. Math. vol. 13 (1929)). (Received September 14, 1950.)

Applied Mathematics

464t. C. N. Mooers: *Automata with learning.* Preliminary report.

The automata moves in an artificial environment having positions or states $q_i(i=1, \cdots, N_q)$. It has a repertory of moves that it can make, each called m_{ij} $(j=1, \cdots, N_i)$. From state q_i by move m_{ij} it goes to a new uniquely determined state q_k, that is, $(q_i, m_{ij})=q_k$. Each state q_i is characterized by an aspect a_i having the value $+1$ or -1. The a_i is a "drive" in the psychological sense, and when a_i is positive the automata is active. In state q_i the automata initially randomly chooses an m_{ij} where all the m's have an equal probability. In the case $(q_i, m_{ij})=q_{i+1}$ whose a_{i+1} is negative (drive extinguished), then the probability is increased for choice m_{ij} when in state q_i. In (q_i, m_{ij}) there is a transfer relation such that when some $m_{i+1,k}$ of q_{i+1} has a probability greater than $2/N_{i+1}$, then the probability of taking m_{ij} in q_i is also increased. The automata as postulated can learn its way through a maze, learning from the goal backwards; it can remember the solution to two or more mazes; it forgets non-used information; and its behavior is not predictable. (Received September 5, 1950.)

465t. L. A. Zadeh: *On stability of linear varying-parameter systems.*

Starting with the definition of stability in the case of linear varying-parameter systems: a system is stable if and only if every bounded input produces a bounded output, it is shown that the necessary and sufficient condition for stability is that the impulsive response of the system $W(t, \tau)$ should belong to $L(0, \infty)$ for all t ($W(t, \tau)$ is the response at t to a unit impulse applied at $t-\tau$). The system function of a linear varying-parameter system is related to $W(t, \tau)$ through $H(s; t)=\int_0^\infty W(t, \tau)e^{-s\tau}d\tau$. From this it follows that the system function of a stable system is analytic in the right half and on the imaginary axis of the s-plane for all t. This result can be applied with advantage to the investigation of stability of linear varying-parameter systems. In particular, it yields useful criteria of stability for differential equations having periodic coefficients. (Received September 14, 1950.)