BOOK REVIEWS

of (1) can be represented as a power series, a contour integral, or as a differential operator. For example, from a formal use of Taylor's theorem it follows that

(2)
$$F(z + y, \alpha) = \sum_{0}^{\infty} y^{n} F(z, \alpha + n)/n!.$$

This formula gives directly a generating function for the sequence $F(z, \alpha+n)/n!$, $n=0, 1, 2, \cdots$, while for z=0 it gives a power series expansion of $F(y, \alpha)$.

As a result of these techniques the author is able to handle problems which could be treated only with difficulty by classical methods. For example, express the Laguerre polynomials in terms of the Legendre polynomials. The scope of the method and the ingenuity of the author are illustrated by the derivation of new, complicated formulas involving the special functions.

An essential point in the study of equation (1) is the proof of the following theorem: Given a bounded sequence $\phi(\alpha+n)$, $n=0, 1, 2, \cdots$, there exists one and only one solution of (1) such that

$$F(z_0, \alpha + n) = \phi(\alpha + n), \qquad n = 0, 1, 2, \cdots$$

This theorem is obtained as a special case of an existence theorem for a general vector difference-differential equation which the author proves. Once the uniqueness is known, it is easy to justify the formal applications such as those in (2).

The author concludes his monograph with some still unsolved questions. One such question is this:

What are the conditions to ensure the existence of a unique solution of the equation

(3)
$$\frac{\partial F(z, \alpha)}{\partial z} = F(z, \alpha - 1)$$

when $\alpha = \alpha_0 + n$, $n = 0, 1, 2, \dots$? If the answer to this were known, some important identities involving number-theoretic functions would be immediate consequences of (3).

Bernard Friedman

Non-linear problems in mechanics of continua. (Proceedings of Symposia in Applied Mathematics, vol. 1.) New York, American Mathematical Society, 1949. 8+219 pp. \$5.25.

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